# Financial Market Integration Asymmetry and Contagion

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**Abstract**

*This paper analyzes extreme joint comovements and contagion effects among equity markets with a focus on the complex dependence structure among multiple markets. Our results reveal an increasing likelihood for extreme comovements among equity markets over the past decades in combination with asymmetric effects between positive and negative comovements. Connectedness among markets during extreme financial times seems to be driven by regionality, even in recent times of globalization. This might be related to information asymmetries for investors regarding distant equity markets. Our results emphasize the importance of trans-regional investments which are necessary to benefit from diversification during financial turmoil.*

**Keywords:** Market integration; contagion; pair-copulas
**JEL classification:** F15; F36; G00; G11; G15

## 1 INTRODUCTION

The degree of equity market integration is crucial for academic and practical questions in the area of finance and is subject to many scientific studies. One reason for this is its importance with respect to asset diversification. Investors can easily access a great variety of different investment opportunities in highly integrated markets which enables higher diversification benefits and more efficient long run investments. On the dark side of market integration stands the counterpart of this taxonomy. In turmoil times, investment risk can only be reduced through diversification if equity markets do not behave in a comonotone manner. However, especially recent

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financial crises give rise to the conjecture that equity markets seem to exhibit strong comovements in bad economic times. This aspect of market integration seems to be crucial for asset allocation and risk management. Beyond that, market integration is connected to many important topics in the financial literature, e.g., building cross-border regulatory regimes, building monetary unions, reducing exchange rate risk or enhancing trade integration.

Even though a common understanding for the meaning of market integration seems to exist, a variety of different methods and measures regarding its quantification can be found. When comparing the literature, several shortcomings related to existing measures of market integration are highlighted. Following these explanations, we argue that a framework for market integration should not rely on linear correlation (see, e.g., Pukthuanthong and Roll, 2009), rather should allow for asymmetric dependencies among equity markets (see, e.g., Longin and Solnik, 2001) and take into account simultaneous movements of multiple equity markets at the same time (see, e.g., Bae et al., 2003; Christiansen and Ranaldo, 2009).

The aim of this paper is to provide such a model framework and analyze several aspects of market integration, i.e., the development for the likelihood of contagion events, the identification of strongly connected economies and the identification of equity markets which are most connected during times of financial turmoil and upswing. We use a pair-copula construction to capture dependencies among equity markets. This helps us to estimate probabilities for comovements of multiple equity markets at the same time. Such estimates are not provided in previous studies because of model framework limitations. In addition, we are able to identify differences regarding dependence structures between various equity markets which is of great importance to identify economies which are strongly connected during extreme financial times. To the best of our knowledge, such an analysis has not been conducted so far and especially the above named research objects have not been addressed in previous studies.

We investigate equity markets over the past decades and find that the likelihood for events in which multiple markets are jointly in a very good or very bad state at the same time increases during our observation period. In addition, we find dependencies in the extreme tails for good and bad market conditions, while the latter seem to be stronger. This leads to asymmetries regarding the occurrence of critical and upswing events and a higher likelihood for equity market crashes than for financial upswings among multiple equity markets. Moreover, a high degree of connectedness during extreme periods is strongly related to geographical distance. For instance the USA exhibit small dependencies during extreme financial times to the remaining economies in a data set which is driven by European countries. Otherwise, strong dependencies during critical as well as upswing market periods can be observed in another data set which is dominated by economies from the American continent. Overall, this shows that connectedness during extreme financial times is driven by regionality. This seems to contradict intuition due to an increasing level of globalization in more recent periods. However, the cause for this observation might still be related to investors' tendency for regional investments and a home bias due to information asymmetries regarding distant equity markets. In our opinion this deepens the understanding of contagion and spillover effects as well as regional and trans-regional diversification effects and therefore plays an important role in the context of market integration.

The remainder of this paper is structured as follows. We provide a literature review in Section 2 and motivate our framework for market integration in Section 3. Section 4 illustrates technical details of our model framework, Section 5 describes quantities applied in the empirical analysis, while Section 6 exhibits results. Section 7 concludes.

## 2 Literature review

A variety of different approaches to measure market integration exists. In general, there seems to be an intuitive understanding what market integration is. Nevertheless, techniques of different analyses regarding market integration often strongly deviate from each other. The focus of this paper is laid on equity market comovements during economic boom and turmoil times. This requires an accurate specification of dependencies among equity markets which is why we mainly present literature that also emphasizes this aspect of market integration.

Longin and Solnik (2001) use an application of extreme value theory to investigate the asymptotic behavior in correlation among equity markets. Within their study they focus on bivariate relations between the US and four other equity markets.

Their main results show that correlation among large losses is significantly higher than among large earnings. This asymmetry can not be reproduced by the multivariate normal model. These results motivate our model framework which is very flexible and capable of reproducing non-linear dependencies. In contrast to their paper, our analysis goes beyond the dimensionality of bivariate relationships as we simultaneously take into account the dependence structure of up to 17 equity markets. In addition, we try to understand and illustrate economic consequences of non-linear dependencies among equity markets.

Bae et al. (2003) examine financial contagion within Asia and Latin America, between these regions and their impact on Europe and the United States. Hereby they empirically assess the number of contemporaneous quantile exceedances among countries and regress these results on regional volatilities as well as interest rate and exchange rate levels. In a second step of their study they also include spillover effects among regions through including the number of contemporaneous coexceedance events in the respective different region within their regression analysis. Their results show that contagion effects are more pronounced in Latin America than in Asia, while Asian contagion has a great impact on the United States. Similar to Bae et al. (2003), one of our interests in this paper is the occurrence of multiple equity markets being in a critical or upswing state at the same time. In contrast to their study, we estimate a parametric model capturing the dependence structure among all markets. Bae et al. (2003) empirically count the number of equity market coexceedances. This restricts the analysis to events which already could have been observed in past data. Furthermore, especially events in which larger numbers of equity markets are in a critical or upswing state at the same time are empirically rare which may lead to high statistical uncertainties when using historical frequencies. Our parametric approach allows the identification of events which did not occur during the observation period and assessing equity market realizations in tail events by simulation which reduces statistical uncertainties. In addition, we set our focus on the development of joint equity market coexceedance behavior over time and focus on the identification of markets which show strong dependencies during extreme financial times.

Kim et al. (2005) and Bartram et al. (2007) both use correlation measures to examine market integration. Kim et al. (2005) analyze market integration among EU member states between 1989 and 2003. To achieve this goal, they estimate time-varying correlations between EU states and a market value weighted EU index with the remaining EU countries and regress these results on variables representing the development of the macroeconomic environment. They find a clear shift in market integration after the introduction of the European Monetary Unit. Bartram et al. (2007) focus on market integration among European equity markets after the introduction of the Euro. They examine time-varying bivariate correlation between European countries and an European equity index. In their analysis they use a combination of a GARCH type model with a Gaussian dependence structure. Their results show that an increase in market dependence after the introduction of the Euro can only be observed for large equity markets, i.e., France, Germany, Italy, Netherlands, and Spain.

Pukthuanthong and Roll (2009) on the other hand criticize the use of linear correlation as a measure for market integration as it appears to be flawed (see Section 3). The authors use a multi-factor model and suggest to use $R^2$ as a measure for market integration. Among 81 countries in their data set, they find an increase in global market integration which is the highest for countries with the longest history of equity markets. However, as shown in more detail in the next section, $R^2$ measures market integration over all states of the economy and does not differentiate between economic conditions. The focus and most important findings in our paper relate to market integration and connectedness during extreme financial times.

Christiansen and Ranaldo (2009) apply the method of Bae et al. (2003) to analyze contagion effects between new and old EU member states. They investigate whether persistence, asset class, volatility and asymmetry effects impact contemporaneous quantile exceedances among countries. In addition they aim to detect whether comovements among EU equity markets change after joining the EU. Overall, their results show strong persistence for coexceedances in new EU member states' equity markets which are linked to old EU member states. Furthermore integration has increased for new EU member states after joining the EU.

Beine et al. (2010) determine coexceedance probabilities for lower and upper return quantiles among pairs of countries and regress these results on financial liberalization, trade integration, industry structure and exchange rate volatility. Their most important findings show financial liberalization only increases left tail comovements among equity markets, while trade integration increases comovements among lower and upper tail events. We also use coexceedance probabilities to quantify the degree of market integration with respect to extreme comovements. In contrast to Beine et al. (2010), we think it is important to take into account the exceedance probability of more than two economies at the same time. Within our analysis, we determine probabilities for up to 12 markets out of 17 being in a critical or upswing state at the same time. We motivate this proceeding in more detail in the next section.

## 3 Motivating a novel framework for market integration

Previous literature shows different ways to measure market integration. Several critical issues for these measures have been stressed which is why we derive attributes that a model framework for market integration should exhibit before we start with our analysis.

First, linear correlation is not an appropriate measure for market integration as it is bounded between attainable minimum and maximum linear correlation and only achieves its maximum borders $\pm 1$ in special cases. This is shown by Pukthuanthong and Roll (2009) in a market integration analysis and by Denuit and Dhaene (2003) in general. Pukthuanthong and Roll (2009) analyze market integration using a multi factor model with principal components as explanatory variables. In an introductory example they show that even though two markets explicitly depend on two global factors and, thus, are perfectly integrated, linear correlation is bounded if factor loadings of explanatory variables are not absolutely proportional. This is true for all linear factor models and not knowing such a relation can lead to wrong conclusions regarding the level of market integration. In general, it can be shown that linear correlation for two random vectors is bounded unless their distinction solely relies on location and scaling parameters. Hence, when using linear correlation for measuring market integration without comparing it to its the upper bound (see Denuit and Dhaene, 2003), deviations regarding the level of market integration might be spurious.

Second, the restriction on symmetric and/or linear dependence structures should be avoided when measuring market integration. Linear models are known to show asymptotic independence in the outer tails of the multivariate distribution. A characteristic like this is not in line with the general consensus of contagion which usually describes events in which turmoil on one equity market leads to economic downturn in other markets. Such a behavior comes along with an increase in dependence during harsh economic conditions and can not be captured by a linear dependence model. In this context Bae et al. (2003) point out that "there is something different about extremely bad events that leads to irrational outcomes, excess volatility, and even panics. In the context of stock returns, this means that if panic grips investors as stock returns fall and leads them to ignore economic fundamentals, one would expect large negative returns to be contagious in a way that small negative returns are not. Correlations that give equal weight to small and large returns are not appropriate for an evaluation of the differential impact of large returns."

In addition, asymmetry between good and bad comovements on equity markets can be observed empirically. Longin and Solnik (2001) examine the relation between the United States and the United Kingdom, France, Germany and Japan on a bivariate basis. They detect correlations between large losses to be significantly higher than among positive returns. Bae et al. (2003) find asymmetric relationships between negative and positive returns on equity markets in Latin America (but not in Asia), while Christiansen and Ranaldo (2009) also detect these asymmetries among old European countries.[1]

To get an idea on the impact of dependence asymmetries for market integration, we consider the following simplified example. We take a look at two hypothetical equity markets and their relation. First, we assume the true scenario to be represented by the left plot in Figure 1. It can be seen that negative comovements seem to be more likely than positive comovements in this scenario. Second, assume we do not know this feature and estimate a linear model for both markets. The estimated model and its reproduction of the true scenario is given in the right plot of Figure 1. For the reason of simplicity, we assume one might measure the level of market integration between both markets through the degree of concordance given by Kendall's $\tau$ for the moment which in contrast to linear correlation is not bounded (see Denuit and Dhaene, 2003).[2] This value is nearly identical for the true scenario (non-linear) and its estimated (linear) version. As a consequence the inference of the same level of

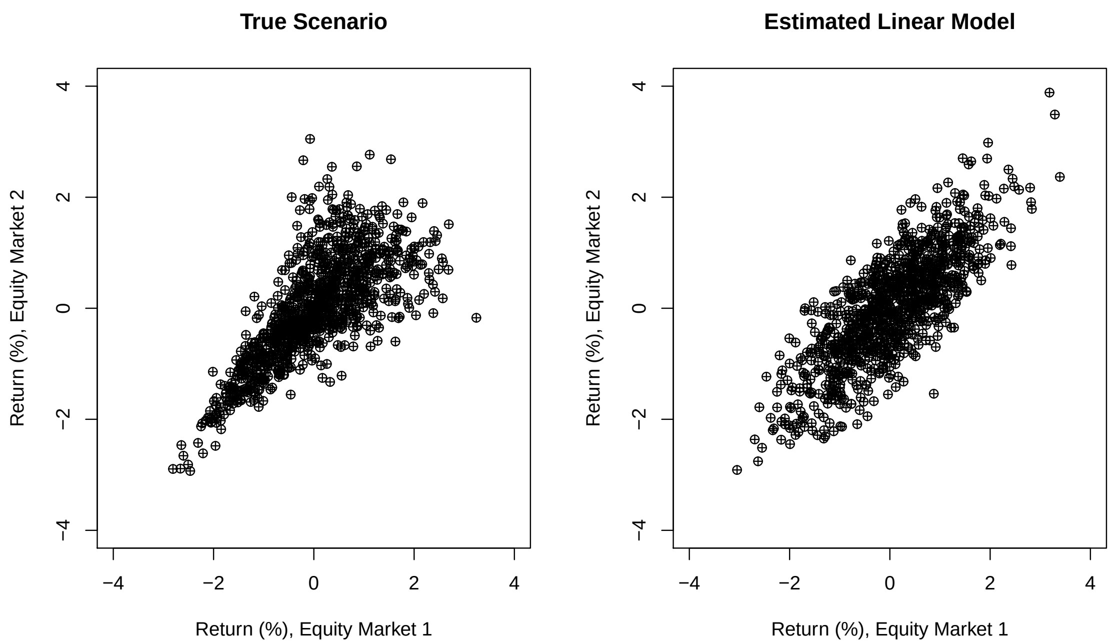
[1] See Christiansen and Ranaldo (2009) for the definition of old European countries.

[2] Kendall's $\tau$ is a rank correlation measure which is based on concordance instead of linear relationships. This attribute makes it possible to compare the degree of dependence between different types of dependence structures.

market integration would be drawn. However, this completely ignores the characteristic of market integration in this case and the fact that, in truth, market integration is mainly driven by negative joint events in both markets. One would also miss this characteristic of market integration when using $R^2$ of the linear regression model as done in Pukthuanthong and Roll (2009). In our example, $R^2 = 0.55$ in the left plot and $R^2 = 0.63$ in the right plot of Figure 1. The reason lies in the linear nature of $R^2$ which is not able to identify the origin for the degree of market integration and the type of dependence structure. While markets seem to be similarly integrated during all economic conditions in the right plot of Figure 1, the level of market integration between both equity markets in the left plot of Figure 1 clearly originates from a strong level of connectedness during bad economic conditions.

This highlights that the characteristic for different types of market integration might be neglected when not accounting for asymmetric dependencies and using integration measures based on linear concepts. We conclude this example with some numbers. Assume, we are interested in the number of joint events in which both markets exhibit a return below its empirical 5%-quantile. While 38 such events occurred among 1,000 realizations, the linear model only estimates 28. On the other hand, the occurrence of joint events in which both market returns exceed their empirical 95%-quantile would be overestimated by the linear model estimating 25 events where only 3 occurred under the true model. This taxonomy could be analogously transfered to a scenario in which market integration is driven by comovements for high returns but reproduced through a linear model. In both cases, the assumption of a linear dependence structure might lead to a wrong conclusion regarding the characteristic of market integration which is why a model framework for market integration should capture non-linear dependencies if they are present in the data.

**Figure 1: Characteristics of market integration in the presence of non-linear dependencies**




Third, a framework for market integration should be able to analyze simultaneous movements of equity markets. A part of the previous literature considers bivariate relations when analyzing market integration (see, e.g., Beine et al., 2010). If the interest lies in quantifying events which affect multiple markets at the same time, bilateral information might not be sufficient. For example, if we are interested in measuring the probability that all markets crash, i.e., each market experiences a critical situation $A_i^C,\ i = 1, ..., n$. This probability is given through

$$P(A_1^C \cap ... \cap A_n^C) = P(A_1^C) \cdot P(A_2^C | A_1^C) \cdot ... \cdot P(A_n^C | A_1^C \cap ... \cap A_{n-1}^C), \qquad (3.1)$$

and we can see that it relies on more information than bilateral (conditional) relationships between markets.[3] The same holds true for multivariate conditional probabilities which might be of special interest to identify the situation on all equity markets given one market experiences a critical situation, e.g.,

$$P(A_1^C \cap ... \cap A_{n-1}^C | A_n^C) = P(A_1^C | A_2^C \cap ... \cap A_{n-1}^C \cap A_n^C) \cdot ... \cdot P(A_{n-1}^C | A_n^C). \qquad (3.2)$$

[3]Except if we face the special cases of abolute dependence or independence.

Furthermore, we think that it is important not to condense information among equity markets for our analysis. One way to do so is given in Bartram et al. (2007). Given one country, they build a stock market index including the remaining equity markets within their data set and estimate time varying correlation between each country and index. Depending on the question of interest, compressing information among equity markets might hinder oneself to identify driving factors for market integration. The following example is made for the illustration of this aspect. We assume three equity markets $EM_1$,$EM_2$,$EM_3$ to follow a multivariate standard Gaussian distribution (strictly for the sake of simplicity) and two possible scenarios regarding the correlation structure among equity markets displayed in Table 1.

**Table 1: Contemporaneous consideration of equity market movements**

| | | | |
|---|---|---|---|
| | Scenario 1 | | |
| | $EM_1$ | $EM_2$ | $EM_3$ |
| $EM_1$ | 1.00 | 0.30 | 0.10 |
| $EM_2$ | 0.30 | 1.00 | 0.70 |
| $EM_3$ | 0.10 | 0.70 | 1.00 |
| | Stress in $EM_1$ | Stress in $EM_2$ | Stress in $EM_3$ |
| Cond. Expected Loss | -0.96 | -1.37 | -1.24 |
| | Scenario 2 | | |
| | $EM_1$ | $EM_2$ | $EM_3$ |
| $EM_1$ | 1.00 | 0.30 | 0.40 |
| $EM_2$ | 0.30 | 1.00 | 0.40 |
| $EM_3$ | 0.40 | 0.40 | 1.00 |
| | Stress in $EM_1$ | Stress in $EM_2$ | Stress in $EM_3$ |
| Cond. Expected Loss | -1.17 | -1.17 | -1.24 |

It can be seen that the average correlation is the same for both scenarios. In addition, e.g., if we calculate an equally weighted index using equity market one and two and determine its correlation to equity market three, we get the same number (0.50) which might lead to the conclusion of identical levels of market integration for equity market number three in both scenarios. However, we can see that the dependence structure clearly differs between both scenarios. A potential economic impact following different dependence structures can be seen when we try to identify the equity market which is most important for contagion. Assuming an equally weighted portfolio of all markets, we calculate conditional expected losses of the portfolio, given one equity market faces a critical event. Here, this means its return (in %) is below its 5%-quantile. In the first scenario, equity market number two is most relevant as we face the highest potential loss if it gets under distress. With a change in the dependence structure this changes, such that equity market number three is potentially most important for negative joint events. Such observations can not be made if each equity market movement is not considered separately.

Summing up, our analysis of market integration should not rely on linear correlation, should be able to capture non-linear dependencies and take into account all market movements at the same time. By this means we try to circumvent restrictions of previous market integration analyses and to provide a framework which is flexible with regards to special features of equity markets' comovements.

## 4 Modeling the multivariate equity return distribution

Our approach takes into account the multivariate distribution of all equity markets under consideration. We use the concept of pair-copulas and therefore separate the information of dependence among equity markets and their marginal distributions. The latter are described first, while the concept of pair copulas is explained subsequently.

### 4.1 Marginal distributions

For each equity index $S_{i,t}$, $i = 1,...,n$, at time $t = 1,..,T$, the log-return is defined through the logarithmic price difference $X_{i,t} = ln(S_{i,t}) - ln(S_{i,t-1})$. Log-returns of financial data are known to show serial correlation, heteroskedasticity, volatility clustering, skewness and often heavy tails. To capture time dynamic effects, we assume log-returns $X_{i,t}$ to follow a stochastic process of an $ARMA(p,q)-GARCH(r,s)$ form given by:

$$X_{i,t} = \mu_{i,t} + \epsilon_{i,t}, \quad \epsilon_{i,t} = \sigma_{i,t} Z_{i,t},$$

where the time dependent drift $\mu_{i,t}$ and variance $\sigma^2_{i,t}$ is defined through

$$\mu_{i,t} = \mu_i + \sum_{j=1}^{p} \phi_{i,j} X_{i,t-1} + \sum_{k=1}^{q} \theta_{i,k} \epsilon_{i,t-1}, \quad \sigma^2_{i,t} = \omega_i + \sum_{j=1}^{r} \alpha_{i,j} \epsilon^2_{i,t-1} + \sum_{k=1}^{s} \beta_{i,k} \sigma^2_{i,t-1}.$$

In addition, residuals $Z_{i,t}$ are assumed to follow a skewed Student t distribution $F_i$ which allows us to capture skewness and leptokurtic behavior of log-returns. After estimating this process for each log-return time series, standardized residuals can be transformed to the unit hypercube using the cumulative distribution function $F_i(Z_{i,t})$ for each estimated marginal distribution. In a next step, these values are used to estimate the dependence structure among equity indices which is modeled through a pair-copula construction. As a consequence, the dependence structure is not driven by any effects of marginal distributions, e.g., serial correlation or volatility clustering, as these effects are filtered through the marginal model before dependencies are estimated.

## 4.2 Dependence modeling using pair-copula constructions

We apply a pair-copula approach to model dependencies between equity markets. The theory of copulas has gained a lot of popularity because it delivers a very flexible concept of dependence modeling. According to the theorem by Sklar (1959), any $n$-dimensional distribution function $F$ can be separated into its univariate marginal distribution functions $F_i,\ i = 1,\dots,n$, and a copula $C$ such that

$$F(x_1,\dots,x_n) = C(F_1(x_1),\dots,F_n(x_n))$$

for all $x_1,\dots,x_n \in \mathbb{R}$. If the marginal distribution functions are continuous, the copula function is even unique. Furthermore the univariate marginal distribution functions do not contain any information about the dependence structure. This information is completely covered by the copula. If $F$ is absolutely continuous and $F_1,\dots,F_n$ are strictly increasing continuous, the density of the $n$-dimensional distribution function is given by

$$f(x_1,\dots,x_n) = \left[\prod_{k=1}^{n} f_k(x_k)\right] \times c(F_1(x_1),\dots,F_n(x_n))$$

where $c$ denotes the density of the copula $C$ and $f$ the marginal densities. Arbitrary univariate marginal distribution functions can be combined with a given copula to define a new multivariate distribution function. Hence, Sklar's theorem provides a powerful method to construct flexible multivariate distribution functions of empirical datasets. There exists a large number of parametric copulas separated into bivariate and multivariate dimensions. In the bivariate case there is a wide variety of different copula types exhibiting flexible and complex dependence patterns, whereas the number of higher dimensional copulas is rather limited. A major reason for this is the fact that not every bivariate copula family can be easily extended to a flexible multivariate case. Concerning multivariate copulas there are various main classes like the one of elliptical copulas, which includes the Gaussian and the Student's t copula. They are always symmetric and the multivariate Gaussian does not posses any tail dependence.[4] The class of multivariate Archimedean copulas, e.g., the Clayton or Gumbel copula, have interesting properties such as tail dependence and can be described through generator functions. Nevertheless, due to exchangeability, the degree of dependence among $x_1,...,x_n$ is identical for each pair if $F$ is modeled through a multivariate Archimedean copula. This seems to be a very serious restriction when modeling financial data. Even though this restriction is circumvented through the use of hierarchical Archimedean copulas, still, a restriction exists as these models only allow to model dependencies which decrease with increasing hierarchal structure.

Dependency patterns of multivariate data are often very complex exhibiting nonlinearity and dependence in the extremes. Therefore in many cases more flexible multivariate copula models are necessary. Another member of the multivariate copula family are pair-copula constructions which constitute a very flexible class of dependence models by sequentially decomposing the joint distributions into bivariate building blocks. It is a treelike construction, built from pair-copulae with conditional distributions as their two arguments. First introduced by Joe (1996), pair-copula constructions were further explored by Bedford and Cooke (2001, 2002) and Kurowicka and Cooke (2006).

We regard a three dimensional case to illustrate a pair-copula construction (see, e.g., Aas et al., 2009; Brechmann and Czado, 2013). The joint density function can be

[4] In simple terms: upper tail dependence describes the asymptotic behavior of a random realization for variable one being very high, given variable two exhibits a high value. Analogously, lower tail dependence can be described.

factorised as

$$f(x_1,x_2,x_3) = f_1(x_1) f_{2|1}(x_2|x_1) f_{3|1,2}(x_3|x_1,x_2). \tag{4.1}$$

For the conditional densities it follows from Sklar's theorem that

$$f_{2|1}(x_2|x_1) = \frac{f_{1,2}(x_1,x_2)}{f_1(x_1)} = \frac{c_{1,2}(F_1(x_1),F_2(x_2)) f_1(x_1) f_2(x_2)}{f_1(x_1)} = c_{1,2}(F_1(x_1),F_2(x_2)) f_2(x_2), \tag{4.2}$$

and

$$\begin{aligned} f_{3|1,2}(x_3|x_1,x_2) &= \frac{f_{2,3|1}(x_2,x_3|x_1)}{f_{2|1}(x_2|x_1)} = \frac{c_{2,3|1}\left(F_{2|1}(x_2|x_1),F_{3|1}(x_3|x_1)\right) f_{2|1}(x_2|x_1) f_{3|1}(x_3|x_1)}{f_{2|1}(x_2|x_1)} \\ &= c_{2,3|1}\left(F_{2|1}(x_2|x_1),F_{3|1}(x_3|x_1)\right) f_{3|1}(x_3|x_1) \\ &\overset{(4.2)}{=} c_{2,3|1}\left(F_{2|1}(x_2|x_1),F_{3|1}(x_3|x_1)\right) c_{1,3}(F_1(x_1),F_3(x_3)) f_3(x_3), \end{aligned} \tag{4.3}$$

with

$$F(x|\boldsymbol{v}) = \frac{\partial C_{xv_j|\boldsymbol{v}_{-j}}\left(F\left(x|\boldsymbol{v}_{-j}\right),F\left(v_j|\boldsymbol{v}_{-j}\right)\right)}{\partial F\left(v_j|\boldsymbol{v}_{-j}\right)}, \tag{4.4}$$

where $C_{xv_j|\boldsymbol{v}_{-j}}$ is a bivariate copula and $\boldsymbol{v}_{-j}$ denotes a vector with the jth component $v_j$ removed. Merging Equations (4.1)-(4.3) leads to

$$\begin{aligned} f(x_1,x_2,x_3) =& f_1(x_1) f_2(x_2) f_3(x_3) c_{1,2}(F_1(x_1),F_2(x_2)) \\ & c_{1,3}(F_1(x_1),F_3(x_3)) c_{2,3|1}\left(F_{2|1}(x_2|x_1),F_{3|1}(x_3|x_1)\right), \end{aligned}$$

which demonstrates how the three-dimensional joint density can be decomposed into a product of bivariate copulas $C_{1,2}$, $C_{1,3}$ and $C_{2,3|1}$.[5] Each bivariate copula can be chosen separately where a great variety of different copula families can be selected, exhibiting various attributes regarding the dependence structure. Thus, pair-copulas are extremely flexible and using the generalization in $n$ dimensions provides a possibility to handle high-dimensional multivariate dependence structures.

Equation 4.1 can be decomposed in different ways which leads to multiple illus-

[5]Note that a common assumption made in this context is the so called simplifying assumption which ensures that the dependence on $X_1$ for $C_{2,3|1}$ solely relates to $F_{2|1}$ and $F_{3|1}$.

trations and varying pair-copula constructions. Thus, selecting the structure of a pair-copula is an important task which exists in addition to the selection of each copula used within the pair-copula and its parameter estimation. A way to illustrate the structure of pair-copulas is given through so called regular vines (see Bedford and Cooke, 2002). According to them a nested set of $n-1$ trees $T_j$, $j = 1,\dots,n-1$, with $n+1-j$ nodes and $n-j$ edges is called $n$-dimensional vine. Each edge of a tree corresponds to a pair-copula density and the edges of tree $T_j$ become the nodes of tree $T_{j+1}$. If two edges in tree $T_j$ share a node the proximity condition requires that the corresponding nodes in tree $T_{j+1}$ are joined by an edge. In the first tree the set of nodes contains all indices $1,\dots,n$, and the set of edges is a set of $n-1$ pairs of these indices. When generating the second tree, it should be noticed that the set of nodes contains now sets of pairs of indices, while the set of edges is built of pairs of pairs of indices and so on. The whole decomposition is defined by the marginal distribution functions and $n(n-1)/2$ bivariate pair-copulas. Note that they do not necessarily need to belong to the same class of copulas.

Two subclasses of regular vines are so called C-vine and D-vine copulas, which are more restrictive cases of regular vine copulas. C-vines use only star like trees and are ordered by importance. Within a C-vine representation those variables exhibiting the highest degrees of dependence are usually modeled in the center of the first trees. Hence, they are interpreted in a way that they are central for the dependence structure which seems to be well suited for the purpose of our analysis as we are interested in identifying equity markets being most connected. Thus, in the following, we focus on the illustration of C-vine constructions. In the heart of the first tree is one central variable which is linked to the remaining variables through bivariate copulas. In the center of the second tree, again, one variable is linked to the remaining variables but now conditional on the pivotal variable in the first tree. This procedure is repeated for the remaining trees whereby each pivotal position is called a root node.

More concrete, the C-vine copula selection is performed in a sequential way as proposed by Czado et al. (2013). For the selection of the first root node the equity market with the highest sum of absolute empirical Kendall's $\tau$ values (to the remaining markets) is chosen. The null hypothesis of independence is tested for each pair of variables. If it is rejected, the appropriate bivariate copula model according to the Akaike Information Criterion (AIC) is selected.[6] In the next step the selection of

[6]The estimation is conducted using the VineCopula package in R. Within this package a number of

the second root node follows. After removing dependence on the first pivotal equity market the equity market with the maximal sum of absolute empirical Kendall's $\tau$ values (among remaining markets) is selected. Again, independence is tested and copula families are selected according to the AIC. The whole selection procedure is then carried forward for all remaining root nodes. In this sense, the selection of all required bivariate copulas forming the building blocks of the C-vine copula is conducted. Estimation then proceeds by joint maximum likelihood estimation over all copula parameters.

## 5 MEASURING MARKET INTEGRATION

A variety of different measures for market integration exist in the literature. While some measures, e.g., correlation, seem to be flawed (see Pukthuanthong and Roll, 2009), the choice for market integration measures may be determined through the focus of the analysis. Similar to Bae et al. (2003), Christiansen and Ranaldo (2009) and Beine et al. (2010) lies our interest in the behavior of equity markets during very good and very bad economic conditions. We first focus on estimating probabilities for coexceedance events on equity markets. In a next step, we are interested in the dependence structure among economies and which are the most connected and closely related during very good and very bad economic times.

Bae et al. (2003) and Christiansen and Ranaldo (2009) both empirically count how many equity markets exhibit positive and negative coexceedances at the same time. Positive means the market exceeds its empirical 95%-quantile and negative, respectively, it falls below the 5%-quantile. They count coexceedances during the whole observation period and, thus, implicitly rely their analysis on the empirical distribution function. We try to extend this approach and estimate probabilities for multiple coexceedance events of markets over time. We proceed as follows. At the end of each year, daily returns from the past four years[7] are used to estimate the

different copula families is available, i.e., the Gauss, Student t, Clayton, Frank, Gumbel, Joe, BB1, BB6, BB7, Tawn type 1 and Tawn type 2 copula as well as rotated versions for the Clayton, Gumbel, Joe, BB1, BB6, BB7, Tawn type 1 and Tawn type 2 copula.

[7]Different time frames were tested during the course of the analysis. Four years seemed to be a good choice regarding the trade-off between higher statistical uncertainties when using less data and the potential bias when using data which include events that occurred a long time ago. Results with different time frames, e.g., one or two years, are available from the authors upon request.

multivariate distribution. Next, we determine the probability

$$P(A^C_{i,t} \cap \ldots \cap A^C_{m,t} \cap A_{m+1,t} \ldots \cap A_{n,t}),\ j = 1,\ldots,n,\ \text{and}\ P(A^U_{i,t} \cap \ldots \cap A^U_{m,t} \cap A_{m+1,t} \ldots \cap A_{n,t}) \tag{5.1}$$

where $i = 1,\ldots,m$ equity markets experience a critical ($A^C_j$) respectively upswing ($A^U_j$) state at the same time $t$, while the remaining markets do not. Critical and upswing events occur if the return for a given market is below or above a certain quantile of the returns' marginal distribution. In contrast to previous approaches, this enables us to quantify occurrence probabilities for events which could not have been observed empirically and to generate more stable estimates with respect to statistical uncertainties. Events in which multiple markets experience a very good or very bad return at the same time are sparse. As a consequence, using empirical estimates for the probability of such events might get unreliable. Otherwise, assuming we correctly estimate the multivariate distribution function among markets, we can use an arbitrary high number of simulation paths which is used for estimating probabilities given in Equation 5.1.

In addition, six measures are used to examine the dependence structure among markets in detail and to identify market integration during extreme economic conditions. We take advantage of the selection and estimation process for C-vine copula structures when analyzing which countries might be highly connected to the others. The equity market which in sum exhibits the highest level of concordance (measured through Kendall's $\tau$) to the remaining markets is determined in the beginning of the selection process. The relationship from this market to remaining markets is estimated using pairs of copulas which defines the first root node. After isolating the dependence from the equity market in the first root node, values for Kendall's $\tau$ among the remaining markets are determined and, again, the market with the highest dependence is chosen as the second root node. This procedure is repeated $n-1$ times. There might be a number of root nodes $s_t \leq (n-1)$ at time $t$, after which independence can not be rejected for any possible pair copula. If such a point is reached, we infer that $s_t$ equity markets are sufficient to model the dependence among $n$ equity markets at time $t$. This might be interpreted similar to a principal component approach with the distinction that dependence drivers are not independent from each other and non-linear dependence among them is possible. Moreover, by means of the position in root nodes, we are able to infer which equity markets are most relevant for modeling the dependence structure among all markets. Following this approach, two quantities are taken into account. The number of markets relevant for dependence $s_t$ and given an equity market is among those $s_t$ markets, we determine its root note position and denote this as the rank $j$ for country $i$ which is important for interconnectedness $IC_{(j),it}$ at time $t$. As already indicated in Section 3, such a proceeding is able to quantify the level of connectedness among markets over all economic conditions, but might fail in identifying which equity markets are most connected during extreme market phases.

This is why we specify four additional measures. First, we determine the expected number of how many remaining markets are in a critical or upswing state, respectively, given equity market $i$ is in such a state at time $t$. We denote this as $CN_{it}^{C}$ for critical states and $CN_{it}^{U}$ for upswing states. Second, we determine the conditional expected return of an equally weighted portfolio, given equity market $i$ is in a critical or upswing state at time $t$, $CL_{it}^{C}$ and $CL_{it}^{U}$. Both measures are determined using simulation techniques after the model is estimated as explained in Section 4. Again, we are interested in the order among equity markets which is why we determine the rank $j$ of each country for these measures at time $t$ which results in $CN_{(j'),it}^{C}$, $CN_{(j'),it}^{U}$, $CL_{(j),it}^{C}$ and $CL_{(j'),it}^{U}$.[8] Contrary to the previous quantities in this paper, these measures allow us to specifically analyze which countries are the most connected during critical and upswing economic conditions.

## 6 Data and empirical results

We start our analysis with a data set of 17 equity indexes representing the economies of Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherlands, Singapore, South Africa, Switzerland, the United Kingdom and the United States. This data set is identical to the cohort data set from Pukthuanthong and Roll (2009) and is chosen because these nations are the largest economies with the longest tradition of free capital mobility (see Pukthuanthong and Roll, 2009). Daily log-returns for each index are taken from Thompson Datastream. Each index is translated into US dollars and values which are identical on two or more successive days are excluded as Datastream fills up data gaps due to holidays

[8]The apostrophe indicates that these value are ordered in decreasing order. See Section 6 for more details.

or comparable events with values from the previous day. In addition, we use one-day lagged values for the US and Canada as European and Asian markets close before North America such that events in both countries impact remaining markets on the following day. Due to the estimation process of the copula model we exclusively use days for estimation on which the log-return for each index is available at the same time. The analysis starts in 1980 and ends in 2015. We assess measures for market integration (see Section 5) at the end of each year and use data from the past four years for the estimation of our model. Thus, our analysis starts in 1984 and covers the development of these measures over the past 32 years. All results which are generated through simulation are based on 200,000 sample paths. During the development of this paper, each analysis has been conducted multiple times. While small deviations could be observed regarding different seeds for random number generation, the general findings are stable.

## 6.1 Probabilities for rare market events over time

Before we illustrate the development of probabilities for coexceedance events, results regarding the estimation of the dependence structure among markets are presented. As stated in Section 4, we use a pair copula model with a canonical vine structure and choose among a variety of different copula families[9] for each pair copula. We compare this flexible C-Vine model to a model with the restriction of exclusively allowing the Gauss copula for each pair copula (restricted model), thus, not taking into account non-linear dependencies. Over the whole time period, the log likelihood is higher for the flexible C-Vine model in comparison to the restricted one. Looking at all copula families from the first root node over all estimated time periods, we find that 85.2% of all estimated copula models exhibit non-linear dependencies, while in 14.0% the Gauss or the Frank copula (not exhibiting tail dependencies) seems to capture dependence best. For the remaining 0.08%, the null hypothesis of independence is not rejected. The first root node seems to be most important regarding the estimation of the dependence structure as the highest degree of dependence is modeled within this node. While the relation between copulas exhibiting tail dependence and not exhibiting tail dependence remains stable for higher root nodes, the fraction of random variable pairs for which the null hypothesis of independence can not be rejected steadily increases. Independence for the half of all random pairs can not be rejected in the sixth root node and after the 14th root node, the null

[9]The choice for the best copula fit is conducted basing on the AIC.

hypothesis of independence can not be rejected in any case. This indicates that the dependence structure among all 17 equity markets can be modeled through up to 14 equity markets in each year. Overall, these results speak for dependence structures divergent to the linear model.

When comparing the fit of both models the AIC of the overall model is taken into account. We find the flexible model dominates the restricted C-vine Gauss model with respect to the AIC as its value is lower in each year. In addition to this comparison, the Vuong (1989) and Clarke (2007) test are applied. Both tests are developed for non-nested models and test the null hypothesis of the flexible C-vine model being indistinguishable from the C-vine Gauss model.[10] In each year the null hypothesis is rejected at the 95% confidence level in case of both tests.[11] Combining the AIC and results for both tests leads to the conclusion of the flexible model being better suited to capture dependencies within our data set than the restricted linear model. Additionally, we analyze if the estimation further improves when using R-vines instead of C-vines for the structure of the pair copula. Even though the AIC slightly decreases through the use of R-vines, the null hypothesis of indistinguishability is never rejected for both tests which is why we stick to the C-vine model. It seems to be a natural choice if one is interested in central drivers for the dependence structure and does not exhibit serious disadvantages regarding the estimation fit in comparison to the even more flexible R-vine structure.

In the next step, we start with the analysis how probabilities for rare events among financial markets evolve over the past 32 years. At first, we need to specify what is considered as a rare event. In accordance with previous literature (see Bae et al., 2003; Beine et al., 2010), we define a single market $i$ to be in a critical state $A^C_{i,t}$ at time $t$, if its log-return is equal or less to its 5%-quantile, analogously, an upswing state $A^U_{i,t}$ is reached if the log-return is equal or higher to its 95%-quantile. Next, we address the question how many markets need to be in a critical or upswing state at the same time to speak of a rare event among all financial markets. Therefore we empirically assess the frequency of comovements among markets over the whole time period. Given this analysis, we define the following rare events: eight (ten, twelve) or more financial markets experience a critical or upswing state on the same day. Empirically, these events occurred with relative frequencies of 2.94% (1.96%,

[10] In our anaylsis, we use the test statistic which is corrected according to Akaike correction.

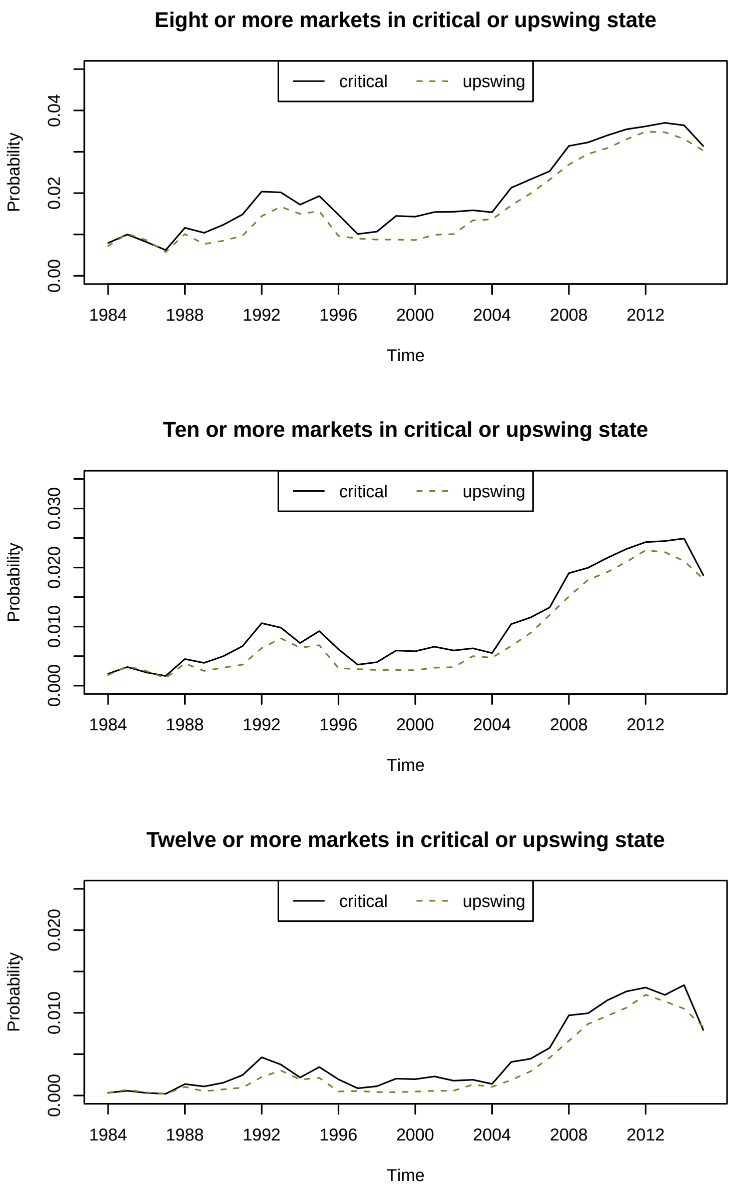
[11] Detailed results on these information are available from the authors upon request.

1.25%) in case of critical comovements and 2.44% (1.60%, 0.71%) in case of upswing comovements and, thus, seem to be reasonably considered as rare. Note that an asymmetric behavior can be observed for these empirical numbers as critical movements on multiple equity markets seem to occur more often than their upswing counterpart.

Figure 2 illustrates results for probabilities of rare comovements among equity markets over time. The black solid line refers to critical market comovements, while joint upswing states on equity markets are represented by the dashed green line. All results are generated using the flexible C-vine model. It can be seen that the occurrence of rare critical as well as upswing events on equity markets is getting more likely over time. For example, the probability of eight or more markets being in a critical state equals 1.43% in the year 2000, while it is 3.54% in the year 2011.

Figure 2 shows that the probability for positive and negative contagion effects among equity markets seems to increase together with the general level of market integration which has been examined in previous analyses. Pukthuanthong and Roll (2009) quantify the level of market integration by means of $R^2$ of a multivariate factor model. Hereby, they detect an increase of market integration for the data set used in our analysis. $R^2$ directs the level of market integration to the whole distribution of equity markets and, by this means, summarizes the information which parts of the distribution are most relevant for the increase in market integration.

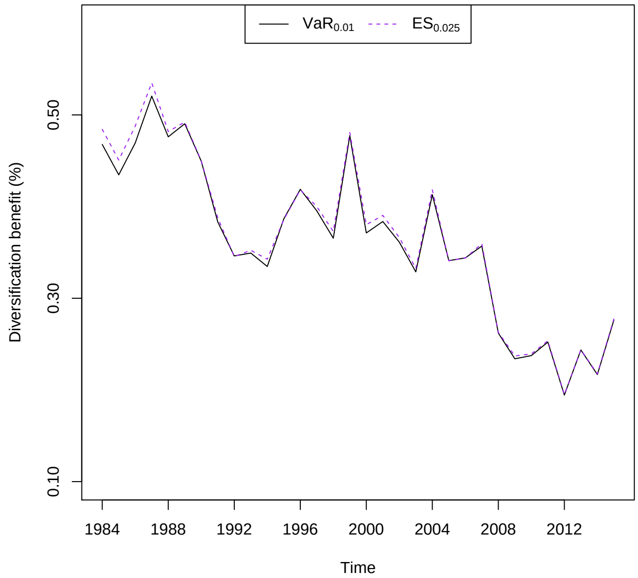
**Figure 2: Probabilities for rare comovements among financial markets over time**




Our analysis allows an isolated view of market integration in the lower and upper tails of the multivariate equity market distribution. We hereby find that market integration has increased in lower tails as well as in upper tails. However, the degree of market integration which is measured through probabilities for rare events seems to be higher for critical than for upswing events among markets over time. This can be ascribed to the existence of non-linear dependencies and asymmetry regarding the degree of upper and lower tail dependencies. To illustrate this more clearly, we estimate the results of Figure 2 with the Gaussian model and compare these estimates with those of the flexible model in Figure 6 which is shown in the appendix. It can be seen that the assessment of probabilities for critical and upswing events through the Gauss model produces lower probability estimates than the flexible model which can be ascribed to lower and upper tail dependencies. In addition, the divergence between the Gauss and the flexible model is higher for probability estimates regarding critical market comovements which speaks for a higher degree of lower tail dependencies and, thus, asymmetry between probability estimates for critical and upswing comovements. The flexible model provides a better fit to the data than the Gaussian model according to the AIC. Moreover, its results, i.e., critical comovements on multiple markets are more likely than upswing comovements and are in line with empirical frequencies for the defined events (2.94%, 1.96%, 1.25% vs. 2.44%, 1.60%, 0.71%). Hence, we infer that in addition to the increase for probabilities regarding positive and negative contagion effects, asymmetry seems to exist between both.

Such a development is important for asset allocation and risk management. If markets show similar reactions during adverse conditions, diversification effects get lost in times in which they are needed the most. To show the economic consequence of this, we assume to hold an equally weighted portfolio of all equity markets over time. To assess the risk of this position, we determine the Value-at-Risk ($VaR_{\alpha,t}$) and the Expected Shortfall ($ES_{\alpha,t}$) under the current estimated multivariate distribution at time $t$ and under the assumption of total dependence among markets. The confidence level of the Value-at-Risk equals 1% and 2.5% for the Expected Shortfall which is motivated by the regulatory market risk framework of Basel III. The difference between the risk measure under total dependence to the risk measure with the current dependence structure is what we ascribe to diversification benefits as this amount can be reduced through diversifying assets and taking advantage of lower dependencies among markets. Figure 3 plots these diversification benefits (in %). It can be seen that regardless of the risk measure, diversification benefits decrease over time. This clearly presents a challenge to portfolio as well as risk managers as portfolio performance decreases and risk management requirements increase. In the light of regulatory developments in the banking and insurance industry, the latter seems to be of special relevance to financial service providers.

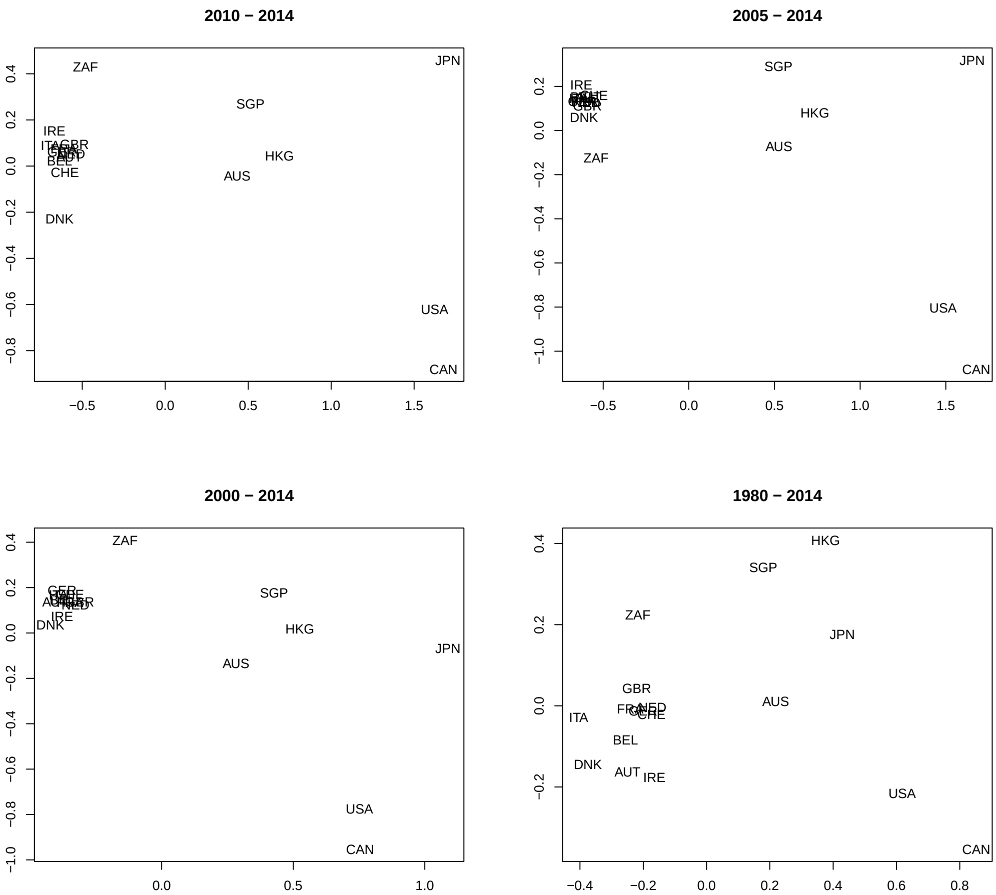
**Figure 3: Diversification effects among equity markets over time**




## 6.2 Identifying countries important for interconnectedness and contagion effects

So far we quantified and analyzed the development of market integration in the outer tails of the multivariate equity market distribution without specifying relevant economies for our observations. In the following part of our analysis, we focus on central markets for the dependence structure among all markets and the identification of highly connected markets during extreme financial times. Starting with the first part of this task, we examine the overall dependence among markets. After removing serial dependence from the equity indices time series (see Section 4.1), we calculate values for Kendall's tau for each pair $\hat{\tau}_{ij}$, $i, j = 1, ..., 17$, $i < j$. In order to illustrate the information of these pairs in a two dimensional space, we apply multidimensional scaling to the dissimilarity measure $1 - \hat{\tau}_{ij}$ (see Brechmann et al., 2013). Results for different time frames are illustrated in Figure 4. The stronger the dependence among equity markets, the closer they are within each plot. The plot in the top left corner shows dependencies for the most recent time frame, while the whole sample period is illustrated in the bottom right. Dependence among equity markets seems to be driven by regionality. For each time frame, European economies are arranged very closely and a great distance to the American economies can be observed, while Asian economies are more close to Australia and also more far away from the European economies. It appears that this seems to be even more pronounced if the time frame is restricted to more recent time periods.

**Figure 4: Multidimensional scaling among equity markets according to dissimilarity measure $1-\hat{\tau}_{ij}$**



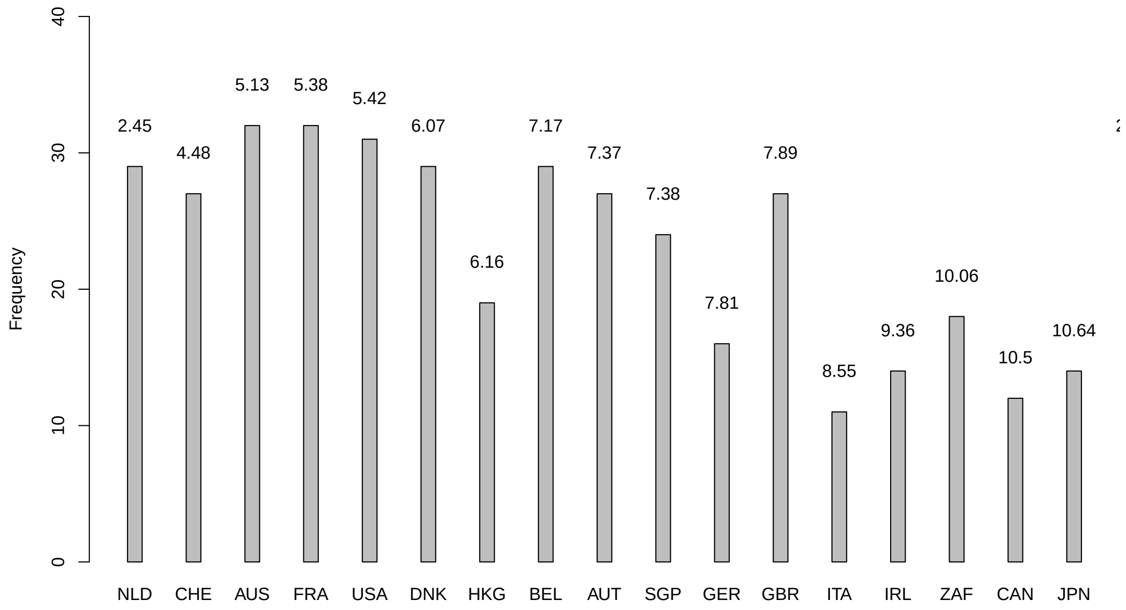

At this point of the analysis, it should be clarified that strong regional linkages measured through Kendall's tau and illustrated as in Figure 4 are not sufficient to examine which markets are strongly linked during extreme financial times, i.e., exhibit tail dependencies. Figure 1 illustrated a model with lower tail dependence and a linear model which both nearly had the same value of Kendall's tau. While dependencies during critical market phases would exist in the first model, critical events would converge to conditional independence in the linear model.[12] Thus, the

[12] See Longin and Solnik (2001) on pages 653 and 654 for a very illustrative explanation of the concept of asymptotic independence for linear dependence models.

results presented in Figure 4 only provide insights regarding the overall degree of dependence among markets and the issue of dependence during extreme financial times is analyzed separately at a later stage in this paper.

Next, we use the selection process of the C-vine model to get a more detailed view on interconnectedness among markets. At each root node, the equity market with the highest degree of dependence to the remaining markets is chosen. Thus, the equity market in the center of the first root node is the highest connected market in the sample at time $t$. Conditioned on the equity market in the first root node, the equity market in the center of the second root node is most connected to the remaining markets and so on. At each point in time, we identify the position in which root node each equity market is selected in the C-vine model. The more in front the position of an equity market, the higher is its general degree of dependence to the remaining markets. As explained in Section 5, after a certain number of root nodes $s_t \leq (n-1)$ the test for independence might be rejected for all remaining pair copulas such that countries in root nodes higher than $s_t$ seem not to be connected to the remaining markets. In this case, countries are not taken into account. Hence, for each equity market at each point in time $t$, we either get the position in the root node of the C-vine structure or the information that this market is not necessary for modeling the dependence structure at time $t$. A summary for this analysis is given in Figure 5. The height of each bar shows how frequently countries are selected among the first $s_t$ root nodes. The number above each bar is the average position each country has over time, given it is among the first $s_t$ root nodes.

**Figure 5: Country specific relevance for interconnectedness among equity markets**




Note: The height of each bar shows how frequently countries are chosen among the first $s$ root nodes over the 32 years. The number above each bar gives the average position each country has over time, given it is among the first $s_t$ root nodes

Looking at Figure 4, we expect European countries to be in front root note positions in the C-Vine copula model. However, this only holds when thinking unconditionally. Therefore, we also illustrate Figure 5 to provide more holistic insights regarding the dependence structure among all markets and how the C-Vine model works. We observe that the Netherlands and Switzerland often seem to be selected and to be in front root note positions. At the same time we can see that Australia and the USA are also selected very often in middle root note positions even though their (unconditional) average values for $\hat{\tau}_{ij}$ are comparably low. How can these results be explained? Imagine we chose the Netherlands in the first root note in the C-Vine model as they exhibit the highest sum of absolute Kendall's tau values for a given time frame. In the next step, one needs to think in conditional dependence. The fraction of dependence which is explained through the Netherlands is higher for European economies than for the remaining economies as their linkage is stronger.

Thus, conditional Kendall's tau values in the second root node among European countries are reduced to a higher extent which might lead to a Non-European economy in the center of the second root note. Table 4 in the Appendix provides detailed results for Figure 5. For all time frames considered, we find a European economy at the first root node position, but at least one Non-European economy among root node position two to four. This illustrates which economies are most central for modeling the whole dependence structure among all markets. Hereby, the Netherlands and Switzerland seem to be most important among European countries and Australia and the USA among Non-European countries.

It might be considered surprising that rather small European economies are selected as central for modeling the dependence structure. It is very important to emphasize that the results in Figure 5 should be interpreted with care and not mixed up with the impact of one country on the others. This is related to the two-sided nature of copulas regarding the description of the relationship between random numbers. Contrary to, e.g., regression analysis in which the relation is one-sided (country $x$ impacts country $y$ and not the other way around) the relation measured through copulas is always two-sided (the relationship can be examined from country $x$ to $y$ as well as from country $y$ to $x$). For example, even though the degree of dependence from country A to country B is high, it does not necessarily mean that country A highly impacts country B. The high degree of dependence could also be originated from a high impact of country B on country A.

In the next step of our analysis, we are aiming to identify which countries are most connected during bad and good economic periods, i.e., in the outer tails of the multivariate distribution. For each equity market we determine the conditional expected number of other markets being in an extreme event, given the equity market itself is in an extreme event at time $t$ ($CN^C_{(j'),it}$, $CN^U_{(j'),it}$). Note that the apostrophe $j'$ indicates that the order is sorted in decreasing manner. In addition, we determine the conditional expected return of an equally weighted portfolio, given equity market $i$ is in a critical or upswing state at time $t$ ($CL^C_{(j),it}$, $CL^U_{(j'),it}$). Both measures are the lowest, the more equity market $i$ is connected in the lower and upper tails of the multivariate distribution. This means, the more equity market $i$ is connected during critical or upswing market phases, the lower is the rank for $CN_{(j'),it}$ as it is sorted in decreasing order. Thus, a low rank implies that in tendency many other markets are in a critical or upswing state, if equity market $i$ faces a critical or upswing state.

The more equity market $i$ is connected during critical or upswing states, the higher portfolio losses and gains should be, if equity market $i$ faces a critical or upswing state. This leads to low rank values for $CL^{C}_{(j),it}$ which is sorted in increasing order and low rank values for $CL^{U}_{(j'),it}$ as it is sorted in decreasing order. Over our period of 32 years, each measure is quantified for each equity market in every year. To subsume these results the average of each measure for each equity market over all years is displayed in Table 2. What immediately can be observed is that the results for the expected conditional number of other markets in extreme events and the ones for conditional expected returns are almost identical. Furthermore, equity markets which are highly connected in critical market phases also seem to be highly connected during economic upturn.

The most important result to us is that connectedness during extreme financial times seems to be driven by regionality. For each measure provided in Table 2 all of the ten European economies are among the first ten positions. As explained previously, our results should not be mixed up with the impact of one equity market on others. Much more, our results exhibit how strong each equity market is linked to the remaining markets whereat it can not be distinguished whether this linkage is based on the impact from the equity market to the other markets or the other way around. In our opinion, this does not diminish our contribution as by means of our model framework and our results, we are able to determine which markets exhibit strong relationships during financial turmoil and economic upturn. Especially such markets are of great relevance with regards to spillover and contagion effects. In our dataset, that is the Netherlands, France, Germany and Switzerland. An information like this seems to be of great value to, e.g., asset and risk managers. Modern asset and risk management techniques are often strongly related to risk measures in the tails of the distribution. Results in Table 2 should be included during portfolio construction as diversification effects during critical financial times might be overestimated if investment decisions are made with a great emphasis on regional more closely related economies. In other words, asset and risk managers probably can take more advantage from trans-regional diversification effects if a reduction of tail risk measures is one of their aims.

**Table 2: Order for equity markets which have the highest average values for $\mathbf{CN^{C}_{(j'),it}}$, $\mathbf{CN^{U}_{(j'),it}}$, $\mathbf{CL^{C}_{(j),it}}$ and $\mathbf{CL^{U}_{(j'),it}}$**

| Country | Avg. Of $CN^{C}_{(j'),it}$ | Country | Avg. Of $CN^{U}_{(j'),it}$ | Country | Avg. Of $CL^{C}_{(j),it}$ | Country | Avg. Of $CL^{U}_{(j'),it}$ |
|---|---|---|---|---|---|---|---|
| Netherlands | 1.8 | Netherlands | 1.8 | Netherlands | 1.7 | Netherlands | 1.7 |
| France | 3.0 | France | 3.3 | France | 3.3 | France | 3.6 |
| Germany | 3.8 | Germany | 3.5 | Germany | 3.8 | Germany | 3.6 |
| Switzerland | 4.0 | Switzerland | 4.1 | Switzerland | 4.0 | Switzerland | 4.2 |
| Belgium | 5.2 | Belgium | 5.4 | Belgium | 5.3 | Belgium | 5.9 |
| United Kingdom | 6.3 | United Kingdom | 6.5 | United Kingdom | 6.3 | United Kingdom | 6.3 |
| Austria | 7.7 | Austria | 7.2 | Austria | 8.0 | Austria | 7.2 |
| Italy | 7.8 | Italy | 7.6 | Italy | 8.1 | Italy | 7.8 |
| Ireland | 8.4 | Ireland | 8.4 | Ireland | 8.5 | Ireland | 8.4 |
| Denmark | 9.8 | Denmark | 10.3 | Denmark | 9.8 | Denmark | 9.6 |
| South Africa | 12.2 | South Africa | 11.9 | South Africa | 11.4 | South Africa | 11.5 |
| Australia | 12.5 | Australia | 12.3 | Australia | 12.4 | Australia | 12.4 |
| Singapore | 13.2 | Singapore | 13.1 | Hong Kong | 12.7 | Hong Kong | 13.2 |
| Hong Kong | 13.3 | Japan | 13.3 | Japan | 13.2 | Japan | 13.2 |
| Japan | 13.4 | Hong Kong | 13.8 | Singapore | 13.3 | Singapore | 13.2 |
| United States | 14.4 | United States | 14.3 | United States | 14.6 | United States | 14.7 |
| Canada | 16.3 | Canada | 16.4 | Canada | 16.7 | Canada | 16.7 |

Note: For each year, the order of countries with the highest expected number of other markets being in a critical or upswing state, given the market itself is in such an event, is determined. An order of one means that the conditional expected value of other countries being in an extreme event at the same time is the highest. Conditional expected returns regarding critical events are sorted in increasing order which means the highest loss is order one regarding critical states. Conditional expected returns regarding upswing events are sorted in decreasing order which means the highest win is order one regarding upswing states. We average these ranks for each country over time. The country with the lowest rank on average is considered to be the most connected during critical and upswing conditions. Detailed results are given in the appendix in Table 5, 6, 7 and 8.

Based on the previous results, we aim to deepen our analysis regarding regional effects of connectedness during extreme financial times. To achieve this goal, we analyze another data set which is dominated by countries from the American continent. We refer to this data set as data set two or the American data set, respectively, in the further course of this analysis. More concrete, we collect data from the United States, Canada, Argentina, Brazil, Mexico, Chile, Peru, Columbia, Australia, South Korea, Japan, France and Germany. Hence, the data set contains thirteen countries and is dominated by eight countries of the American continent. Three countries are from Asia and Australia, while only two European economies are included in the data set. Again, we use country specific indices provided by Thompson Datastream. Due to limited data availability of some countries, our time frame spans from 1995 until 2015 such that first results start in 1999 when using the four year backward looking point of view for the estimation of our model. Again, one day lagged data values are used between American and European as well as Asian countries.

We start our comparison between the European and American data set with estimated probabilities for comovements among markets. Results are displayed in Figure 9 in the Appendix. As data set two only includes thirteen countries instead of 17, we focus on events in which six, eight or ten markets, respectively, are in a critical or upswing state at the same time. The magnitude of estimated probabilities is roughly comparable to the results of the European data set by this means. We find that results behave similar, such that probabilities for extreme financial events increase over time. This speaks for a higher level of connectedness during extreme market phases in more recent times. In addition, it seems that asymmetry occurs between critical and upswing events which speaks for stronger lower tail than upper dependence among the economies of interest.

In a next step, we examine the overall level of connectedness among countries and economies central for the multivariate dependence structure. Analogously to Figure 4 and 5, Figure 7 and 8 in the Appendix illustrate results for data set two. Again, we find that economies with lower geographical distances exhibit higher dependencies. For each time frame, we find that American economies are located more closely to each other in Figure 7 and the European economies are situated more far away in the plot. Regarding the results in Figure 8, independence can not be rejected after a number of up to ten countries in each year. As already explained, countries are selected according to Kendall's Tau within each root node which captures the overall level of connectedness without differentiating for the type of dependence (lower, upper or no tail dependence). The further an economy is selected in root notes, the higher is its overall level of connectedness. We find that the USA and Brazil are usually chosen at early positions during the estimation process which speaks for a high degree of dependence of these economies to the remaining ones. Even if France and Germany are not chosen as often, we can see that they seem to exhibit a relatively high level of conditional dependence to the remaining counties. This is similar to the results in Figure 5. Table 3 provides a summary for results of data set two regarding the four measures which we already use in Table 2. Again, we find strong similarities to the findings of the European data set. This means that six out of eight American countries are among the first eight positions for each measure in Table 2. In contrast to the European data set in which the USA is on second last position, it is on the first and second position regarding all measures in Table 2. It appears, that the USA, Mexico, Brazil and Canada seem to be most connected during extreme financial times in the American data set. These results further strengthen our conjecture that connectedness during extreme financial times is driven by regionality.

The relevance and importance of these results is enhanced by previous findings in the literature which detects decreasing capital flows of asset trade for countries with higher geographical distances and preferences of mutual fund managers for local investments, respectively (see Gârleanu et al., 2015). These kinds of behavior usually are explained by information asymmetries increasing with local distance which means that trans-regional investments are often hindered by costs necessary to overcome such information asymmetry. Regarding the topic of our paper, this means that investments are usually made in geographical closely related regions. At the same time these regions usually are collectively hit by critical or upswing events due to high dependencies during such events. By this means, the importance for overcoming trans-regional information asymmetries seems to be a necessary task in order to benefit from diversification benefits during extreme financial times.

**Table 3: Order for equity markets (data set two) which have the highest average values for $\mathbf{CN^{C}_{(j'),it}}$, $\mathbf{CN^{U}_{(j'),it}}$, $\mathbf{CL^{C}_{(j),it}}$ and $\mathbf{CL^{U}_{(j'),it}}$**

| Country | Avg. Of $CN^{C}_{(j'),it}$ | Country | Avg. Of $CN^{U}_{(j'),it}$ | Country | Avg. Of $CL^{C}_{(j),it}$ | Country | Avg. Of $CL^{U}_{(j'),it}$ |
|---|---|---|---|---|---|---|---|
| United States | 1.7 | United States | 1.5 | Brazil | 2.1 | United States | 1.7 |
| Brazil | 3.1 | Mexico | 2.8 | United States | 2.4 | Brazil | 2.3 |
| Mexico | 3.2 | Brazil | 3.3 | Mexico | 3.2 | Mexico | 2.8 |
| Canada | 3.8 | Canada | 4.3 | Canada | 3.9 | Canada | 4.9 |
| Australia | 5.8 | Australia | 5.5 | Australia | 5.9 | Argentina | 6.2 |
| Argentina | 7.0 | Argentina | 7.6 | Argentina | 6.1 | Australia | 6.4 |
| Chile | 7.4 | Chile | 7.8 | Chile | 7.6 | South Korea | 7.0 |
| Peru | 8.4 | France | 8.9 | South Korea | 8.1 | Chile | 8.2 |
| South Korea | 9.3 | South Korea | 8.9 | Peru | 8.2 | Peru | 8.9 |
| Japan | 9.8 | Japan | 9.0 | Japan | 10.1 | Japan | 9.2 |
| France | 9.8 | Peru | 9.2 | France | 10.8 | France | 10.4 |
| Germany | 10.0 | Germany | 10.2 | Germany | 11.1 | Germany | 11.0 |
| Columbia | 11.6 | Columbia | 12.1 | Columbia | 11.6 | Columbia | 11.9 |

Note: For each year, the order of countries with the highest expected number of other markets being in a critical or upswing state, given the market itself is in such an event, is determined. An order of one means that the conditional expected value of other countries being in an extreme event at the same time is the highest. Conditional expected returns regarding critical events are sorted in increasing order which means the highest loss is order one regarding critical states. Conditional expected returns regarding upswing events are sorted in decreasing order which means the highest win is order one regarding upswing states. We average these ranks for each country over time. The country with the lowest rank on average is considered to be the most connected during critical and upswing conditions. Detailed results are given in the appendix in Table 9, 10, 11 and 12.

## 7 Conclusion

This paper analyzes the development of market integration for a variety of equity markets. More specifically, our main interest lies on the likelihood of rare comovements among markets over time, thereby focusing on market integration in the outer tails of the multivariate equity market distribution. In addition, we identify which markets are central for the dependence structure among all markets and which markets are most connected during financial turmoil and upswing.

We find that equity markets seem to exhibit higher dependencies in the lower tail of their distribution than in the upper tail. This leads to higher probabilities regarding negative contagion effects than for positive contagion effects. Overall, probabilities for rare comovements among equity markets increased for the past decades which reduces diversification among markets and intensifies asset allocation as well as risk management problems.

Furthermore, we observe that connectedness during extreme financial times seems to be strongly linked to geographical distance even in recent times of increasing globalization. For instance, while the USA appears to be weakly connected to the European economies during critical and upswing market phases, it appears to be strongly connected to geographically more close economies in such times. Analogous results can be observed for Germany and France in data sets driven by European and American economies, respectively. Overall, we identify the Netherlands, France, Germany and Switzerland to be most connected during extreme financial times in a data set including a majority of European equity markets and the USA, Brazil, Mexico and Canada in a data set including a majority of American equity markets. These results underline that connectedness during financial turmoil and upswing is driven by regionality. Such observations might be linked to regional investment preferences by investors due to information asymmetries for further distant equity markets. Thus, we find great importance in overcoming such information asymmetries in order to benefit from trans-regional diversification benefits.

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# A Appendix

**Figure 6: Probabilities for rare comovements among financial markets over time - linear vs. non-linear model**

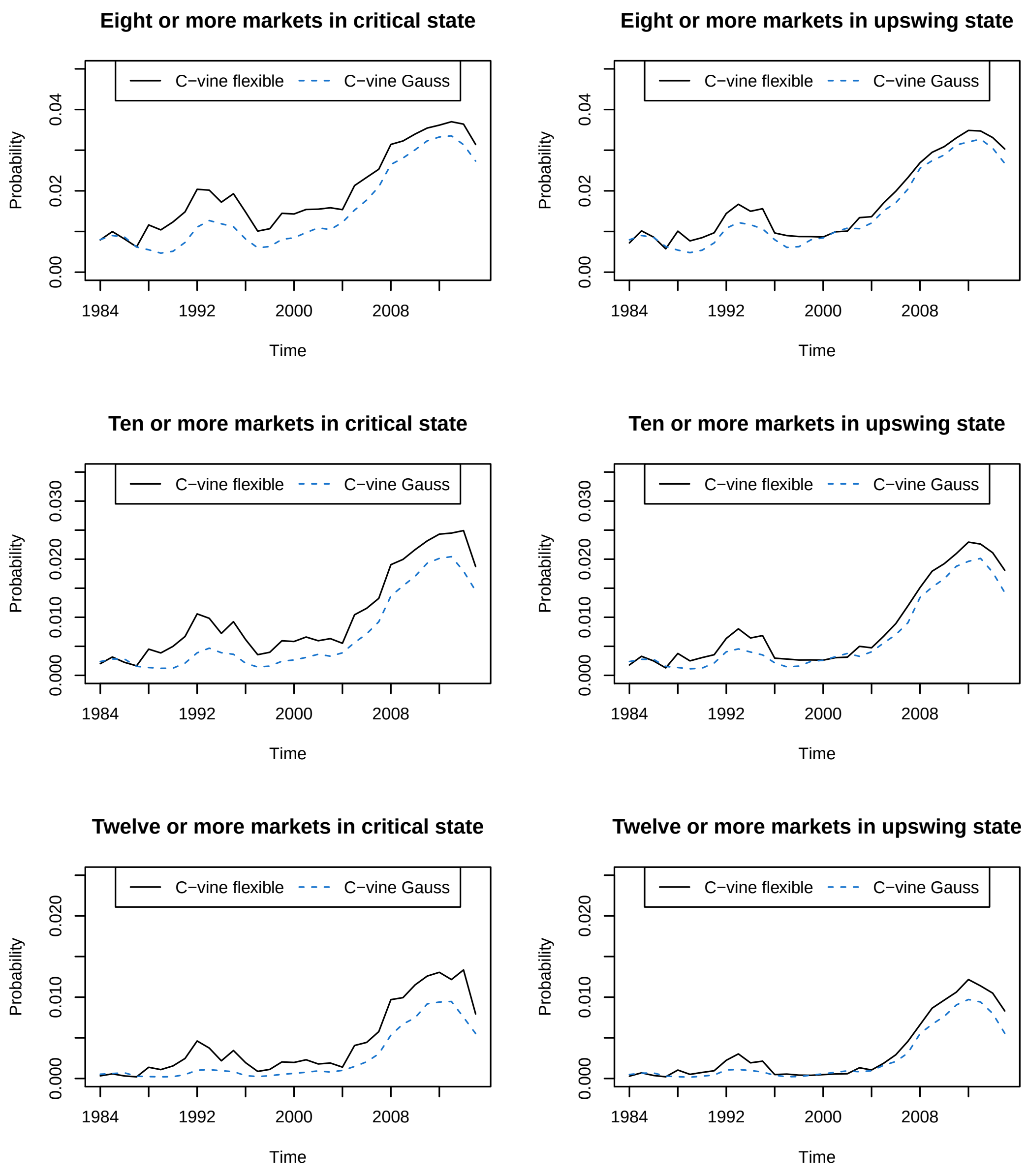


**Table 4: Detailed results regarding Figure 5**

| Year | AUS | AUT | BEL | CAN | DNK | FRA | GER | HKG | IRE | ITA | JPN | NLD | SGP | ZAF | CHE | GBR | USA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1984 | 4 | 3 | 7 | - | 5 | 9 | - | 6 | - | - | 11 | 2 | - | - | 1 | 8 | 10 |
| 1985 | 4 | 3 | 9 | 12 | 5 | 7 | - | 6 | - | - | 10 | 2 | 11 | - | 1 | 8 | - |
| 1986 | 4 | 5 | 9 | - | 3 | 7 | - | - | 11 | - | 10 | 2 | 8 | - | 1 | 6 | 12 |
| 1987 | 4 | 8 | 5 | - | 2 | 12 | - | - | 6 | 10 | - | 3 | 7 | - | 1 | 11 | 9 |
| 1988 | 4 | - | 5 | - | 2 | 10 | - | - | 7 | 9 | - | 3 | 8 | 11 | 1 | - | 6 |
| 1989 | 5 | 13 | 12 | 11 | 3 | 9 | - | - | 6 | 8 | - | 2 | 7 | 10 | 1 | - | 4 |
| 1990 | 10 | - | 9 | 11 | 3 | 8 | - | - | 7 | 12 | - | 2 | 5 | 6 | 1 | - | 4 |
| 1991 | 8 | 7 | 12 | - | 3 | 6 | - | - | - | - | 9 | 2 | 5 | 10 | 1 | 11 | 4 |
| 1992 | 9 | 7 | 6 | - | 3 | 8 | - | 11 | - | 12 | - | 2 | 4 | 13 | 1 | 10 | 5 |
| 1993 | 9 | 8 | 6 | - | 3 | 7 | - | 12 | 10 | - | - | 1 | 4 | - | 2 | 11 | 5 |
| 1994 | 9 | 8 | 2 | - | 6 | 7 | 11 | - | 10 | - | - | 1 | 3 | 12 | 4 | - | 5 |
| 1995 | 3 | 7 | 2 | 10 | - | 6 | 12 | - | 8 | - | - | 1 | 9 | - | 4 | 11 | 5 |
| 1996 | 8 | - | 7 | 11 | 3 | 12 | 2 | - | - | - | 9 | 1 | 4 | - | 10 | 5 | 6 |
| 1997 | 8 | 10 | 7 | 11 | 3 | 9 | 2 | - | - | - | - | 1 | 4 | 12 | - | 5 | 6 |
| 1998 | 6 | 10 | 7 | 12 | 3 | 9 | 2 | - | - | - | - | 1 | 8 | 11 | - | 5 | 4 |
| 1999 | 8 | 10 | 7 | 12 | 4 | 9 | 2 | - | - | - | - | 1 | 6 | 11 | - | 5 | 3 |
| 2000 | 8 | - | 4 | - | 7 | 10 | 2 | 6 | - | - | - | 1 | 11 | - | 9 | 5 | 3 |
| 2001 | 7 | 10 | 13 | 12 | 6 | 5 | 8 | 2 | - | - | - | 1 | 11 | - | 3 | 9 | 4 |
| 2002 | 7 | 9 | 12 | - | 6 | 3 | 10 | 4 | - | - | - | 1 | 11 | - | 5 | 8 | 2 |
| 2003 | 9 | 6 | 13 | - | 8 | 3 | - | 5 | - | 12 | 11 | 1 | - | 7 | 4 | 10 | 2 |
| 2004 | 8 | 6 | 3 | - | - | 1 | 13 | 4 | 12 | 10 | - | 0 | 11 | 9 | 5 | 7 | 2 |
| 2005 | 2 | 7 | 3 | - | 12 | 1 | 9 | 11 | - | - | - | 0 | 5 | 10 | 6 | 8 | 4 |
| 2006 | 2 | 7 | 5 | - | - | 1 | 10 | 13 | 12 | 3 | - | 0 | 6 | 9 | 11 | 8 | 4 |
| 2007 | 2 | - | 5 | - | 7 | 1 | - | 4 | 10 | 3 | 11 | 8 | - | 9 | 12 | - | 6 |
| 2008 | 2 | 11 | 3 | - | 8 | 1 | - | 4 | 12 | 5 | 9 | 10 | - | - | - | 6 | 7 |
| 2009 | 2 | 3 | 4 | - | 9 | 1 | - | 5 | 13 | 10 | 11 | 8 | - | 12 | - | 6 | 7 |
| 2010 | 2 | 5 | 9 | 6 | 13 | 1 | 10 | 4 | - | - | - | 3 | 11 | - | 7 | 8 | 12 |
| 2011 | 2 | 5 | 13 | 6 | 9 | 1 | 10 | 4 | - | - | 14 | 3 | - | 12 | 7 | 8 | 11 |
| 2012 | 2 | 6 | 9 | - | 12 | 1 | 11 | 5 | - | - | 13 | 3 | - | 10 | 7 | 8 | 4 |
| 2013 | 2 | 7 | - | - | 9 | 1 | 11 | 5 | - | - | 10 | 3 | 12 | 0 | 6 | 8 | 4 |
| 2014 | 2 | 9 | - | 12 | 8 | 3 | - | 6 | - | - | 11 | 1 | - | 7 | 5 | 10 | 4 |
| 2015 | 2 | 9 | - | - | 11 | 3 | - | - | 7 | - | 10 | 1 | 6 | - | 5 | 8 | 4 |
| Sum among root notes | 32 | 27 | 29 | 12 | 29 | 32 | 16 | 19 | 14 | 11 | 14 | 29 | 24 | 18 | 27 | 27 | 31 |
| Conditional mean for $IC_{(j),it}$ | 5,13 | 7,37 | 7,17 | 10,5 | 6,07 | 5,38 | 7,81 | 6,16 | 9,36 | 8,55 | 10,64 | 2,45 | 7,38 | 10,06 | 4,48 | 7,89 | 5,42 |

**Table 5: Rank of each country in each year for $\mathbf{CN^{C}_{(j'),it}}$**

| Year | AUS | AUT | BEL | CAN | DNK | FRA | GER | HKG | IRE | ITA | JPN | NLD | SGP | ZAF | CHE | GBR | USA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1984 | 12 | 3 | 10 | 17 | 5 | 7 | 2 | 14 | 11 | 13 | 8 | 4 | 15 | 9 | 1 | 6 | 16 |
| 1985 | 11 | 3 | 7 | 17 | 10 | 5 | 2 | 15 | 12 | 13 | 8 | 4 | 16 | 9 | 1 | 6 | 14 |
| 1986 | 10 | 6 | 7 | 17 | 8 | 4 | 2 | 16 | 9 | 13 | 11 | 3 | 15 | 12 | 1 | 5 | 14 |
| 1987 | 11 | 7 | 8 | 17 | 6 | 9 | 3 | 14 | 10 | 13 | 5 | 2 | 16 | 12 | 1 | 4 | 15 |
| 1988 | 12 | 10 | 8 | 17 | 4 | 5 | 7 | 16 | 6 | 13 | 9 | 2 | 15 | 11 | 1 | 3 | 14 |
| 1989 | 13 | 11 | 3 | 17 | 6 | 4 | 8 | 15 | 7 | 10 | 9 | 2 | 12 | 16 | 1 | 5 | 14 |
| 1990 | 14 | 10 | 3 | 17 | 7 | 5 | 4 | 16 | 6 | 11 | 9 | 2 | 13 | 15 | 1 | 8 | 12 |
| 1991 | 15 | 9 | 5 | 17 | 11 | 2 | 4 | 14 | 7 | 6 | 12 | 3 | 10 | 16 | 1 | 8 | 13 |
| 1992 | 16 | 8 | 5 | 17 | 11 | 4 | 2 | 12 | 7 | 6 | 13 | 3 | 10 | 14 | 1 | 9 | 15 |
| 1993 | 15 | 7 | 5 | 16 | 10 | 4 | 3 | 14 | 6 | 9 | 11 | 1 | 12 | 13 | 2 | 8 | 17 |
| 1994 | 12 | 6 | 5 | 17 | 15 | 3 | 4 | 14 | 8 | 9 | 11 | 1 | 10 | 13 | 2 | 7 | 16 |
| 1995 | 14 | 6 | 5 | 16 | 10 | 2 | 4 | 13 | 8 | 9 | 15 | 1 | 11 | 12 | 3 | 7 | 17 |
| 1996 | 13 | 6 | 5 | 16 | 9 | 4 | 3 | 12 | 7 | 10 | 17 | 1 | 11 | 14 | 2 | 8 | 15 |
| 1997 | 14 | 7 | 4 | 12 | 9 | 5 | 3 | 10 | 6 | 16 | 17 | 1 | 15 | 13 | 2 | 8 | 11 |
| 1998 | 10 | 7 | 3 | 14 | 13 | 5 | 2 | 9 | 6 | 15 | 17 | 1 | 16 | 11 | 4 | 8 | 12 |
| 1999 | 11 | 6 | 4 | 16 | 17 | 5 | 2 | 10 | 7 | 12 | 14 | 1 | 15 | 9 | 3 | 8 | 13 |
| 2000 | 11 | 6 | 5 | 14 | 17 | 3 | 1 | 9 | 10 | 8 | 15 | 2 | 16 | 12 | 4 | 7 | 13 |
| 2001 | 10 | 8 | 7 | 15 | 17 | 2 | 3 | 9 | 11 | 5 | 14 | 1 | 13 | 12 | 4 | 6 | 16 |
| 2002 | 13 | 9 | 4 | 16 | 12 | 2 | 3 | 8 | 10 | 5 | 15 | 1 | 17 | 11 | 6 | 7 | 14 |
| 2003 | 12 | 10 | 6 | 17 | 9 | 2 | 5 | 11 | 8 | 4 | 16 | 1 | 14 | 15 | 3 | 7 | 13 |
| 2004 | 14 | 10 | 7 | 17 | 9 | 2 | 5 | 11 | 8 | 3 | 15 | 1 | 13 | 16 | 6 | 4 | 12 |
| 2005 | 12 | 11 | 4 | 15 | 9 | 1 | 7 | 16 | 8 | 3 | 17 | 2 | 14 | 10 | 6 | 5 | 13 |
| 2006 | 11 | 10 | 4 | 16 | 9 | 1 | 6 | 17 | 8 | 3 | 15 | 2 | 13 | 14 | 5 | 7 | 12 |
| 2007 | 11 | 10 | 4 | 17 | 8 | 1 | 5 | 16 | 9 | 3 | 14 | 2 | 13 | 12 | 7 | 6 | 15 |
| 2008 | 13 | 9 | 3 | 17 | 6 | 1 | 5 | 15 | 10 | 4 | 14 | 2 | 12 | 11 | 8 | 7 | 16 |
| 2009 | 13 | 9 | 6 | 17 | 7 | 1 | 4 | 14 | 10 | 3 | 15 | 2 | 12 | 11 | 8 | 5 | 16 |
| 2010 | 13 | 8 | 6 | 17 | 9 | 1 | 4 | 14 | 10 | 3 | 15 | 2 | 12 | 11 | 7 | 5 | 16 |
| 2011 | 13 | 7 | 6 | 17 | 9 | 1 | 4 | 14 | 10 | 3 | 15 | 2 | 11 | 12 | 8 | 5 | 16 |
| 2012 | 13 | 7 | 6 | 17 | 10 | 1 | 4 | 14 | 9 | 3 | 15 | 2 | 12 | 11 | 8 | 5 | 16 |
| 2013 | 12 | 8 | 4 | 17 | 10 | 2 | 3 | 14 | 7 | 5 | 15 | 1 | 13 | 11 | 9 | 6 | 16 |
| 2014 | 12 | 6 | 4 | 16 | 10 | 1 | 3 | 14 | 9 | 8 | 17 | 2 | 13 | 11 | 5 | 7 | 15 |
| 2015 | 13 | 7 | 4 | 16 | 10 | 2 | 3 | 14 | 9 | 8 | 17 | 1 | 12 | 11 | 6 | 5 | 15 |
| Mean | 12.5 | 7.7 | 5.2 | 16.3 | 9.8 | 3.0 | 3.8 | 13.3 | 8.4 | 7.8 | 13.4 | 1.8 | 13.2 | 12.2 | 4.0 | 6.3 | 14.4 |
| Order of mean | 12 | 7 | 5 | 17 | 10 | 2 | 3 | 14 | 9 | 8 | 15 | 1 | 13 | 11 | 4 | 6 | 16 |

**Table 6: Rank of each country in each year for $\mathbf{CN^{U}_{(j'),it}}$**

| Year | AUS | AUT | BEL | CAN | DNK | FRA | GER | HKG | IRE | ITA | JPN | NLD | SGP | ZAF | CHE | GBR | USA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1984 | 9 | 3 | 11 | 17 | 12 | 6 | 1 | 14 | 10 | 16 | 8 | 4 | 13 | 7 | 2 | 5 | 15 |
| 1985 | 10 | 3 | 7 | 17 | 11 | 8 | 2 | 15 | 12 | 16 | 9 | 4 | 13 | 5 | 1 | 6 | 14 |
| 1986 | 8 | 6 | 7 | 17 | 9 | 5 | 2 | 16 | 12 | 14 | 11 | 3 | 13 | 10 | 1 | 4 | 15 |
| 1987 | 11 | 5 | 6 | 17 | 10 | 8 | 4 | 16 | 9 | 13 | 7 | 2 | 12 | 14 | 1 | 3 | 15 |
| 1988 | 13 | 9 | 8 | 16 | 5 | 3 | 4 | 17 | 6 | 11 | 10 | 2 | 14 | 12 | 1 | 7 | 15 |
| 1989 | 13 | 11 | 8 | 15 | 4 | 3 | 7 | 17 | 6 | 9 | 10 | 2 | 16 | 12 | 1 | 5 | 14 |
| 1990 | 12 | 11 | 6 | 16 | 7 | 4 | 3 | 13 | 5 | 8 | 9 | 2 | 17 | 15 | 1 | 10 | 14 |
| 1991 | 12 | 8 | 4 | 17 | 10 | 5 | 3 | 16 | 7 | 6 | 11 | 2 | 13 | 15 | 1 | 9 | 14 |
| 1992 | 13 | 8 | 5 | 17 | 10 | 4 | 3 | 14 | 7 | 6 | 12 | 2 | 11 | 15 | 1 | 9 | 16 |
| 1993 | 14 | 7 | 3 | 17 | 10 | 4 | 5 | 15 | 6 | 8 | 11 | 1 | 12 | 13 | 2 | 9 | 16 |
| 1994 | 13 | 6 | 4 | 17 | 15 | 5 | 3 | 11 | 8 | 9 | 14 | 1 | 10 | 12 | 2 | 7 | 16 |
| 1995 | 12 | 4 | 5 | 17 | 9 | 6 | 3 | 14 | 8 | 11 | 15 | 1 | 10 | 13 | 2 | 7 | 16 |
| 1996 | 14 | 5 | 4 | 17 | 9 | 6 | 2 | 11 | 7 | 13 | 16 | 1 | 10 | 12 | 3 | 8 | 15 |
| 1997 | 11 | 2 | 5 | 17 | 9 | 7 | 3 | 13 | 6 | 14 | 16 | 1 | 10 | 12 | 4 | 8 | 15 |
| 1998 | 17 | 6 | 2 | 13 | 11 | 5 | 3 | 16 | 7 | 15 | 10 | 1 | 12 | 14 | 4 | 8 | 9 |
| 1999 | 17 | 6 | 2 | 13 | 16 | 5 | 3 | 11 | 8 | 9 | 14 | 1 | 15 | 10 | 4 | 7 | 12 |
| 2000 | 14 | 8 | 4 | 15 | 17 | 2 | 3 | 12 | 9 | 7 | 13 | 1 | 16 | 11 | 5 | 6 | 10 |
| 2001 | 13 | 8 | 5 | 14 | 17 | 2 | 3 | 12 | 9 | 6 | 16 | 1 | 15 | 11 | 4 | 7 | 10 |
| 2002 | 14 | 8 | 7 | 16 | 15 | 2 | 3 | 11 | 9 | 4 | 17 | 1 | 13 | 10 | 5 | 6 | 12 |
| 2003 | 13 | 9 | 7 | 17 | 14 | 2 | 4 | 10 | 8 | 3 | 16 | 1 | 15 | 12 | 5 | 6 | 11 |
| 2004 | 13 | 10 | 6 | 16 | 9 | 1 | 5 | 12 | 8 | 3 | 15 | 2 | 14 | 17 | 7 | 4 | 11 |
| 2005 | 11 | 10 | 5 | 17 | 8 | 1 | 6 | 12 | 9 | 3 | 14 | 2 | 13 | 15 | 4 | 7 | 16 |
| 2006 | 11 | 10 | 4 | 17 | 8 | 1 | 6 | 15 | 9 | 3 | 13 | 2 | 14 | 12 | 5 | 7 | 16 |
| 2007 | 11 | 10 | 5 | 17 | 8 | 1 | 4 | 15 | 9 | 3 | 14 | 2 | 13 | 12 | 6 | 7 | 16 |
| 2008 | 12 | 8 | 7 | 17 | 10 | 1 | 3 | 14 | 9 | 4 | 15 | 2 | 13 | 11 | 5 | 6 | 16 |
| 2009 | 12 | 8 | 5 | 17 | 10 | 1 | 4 | 14 | 9 | 3 | 15 | 2 | 13 | 11 | 7 | 6 | 16 |
| 2010 | 12 | 8 | 6 | 17 | 9 | 1 | 4 | 14 | 10 | 3 | 16 | 2 | 13 | 11 | 7 | 5 | 15 |
| 2011 | 12 | 7 | 5 | 17 | 9 | 1 | 4 | 14 | 10 | 3 | 16 | 2 | 13 | 11 | 8 | 6 | 15 |
| 2012 | 12 | 7 | 6 | 17 | 10 | 1 | 4 | 14 | 9 | 3 | 15 | 2 | 13 | 11 | 8 | 5 | 16 |
| 2013 | 12 | 7 | 6 | 17 | 10 | 1 | 3 | 14 | 9 | 4 | 15 | 2 | 13 | 11 | 8 | 5 | 16 |
| 2014 | 11 | 5 | 4 | 17 | 10 | 2 | 3 | 14 | 9 | 6 | 15 | 1 | 13 | 12 | 8 | 7 | 16 |
| 2015 | 12 | 6 | 4 | 17 | 10 | 2 | 3 | 14 | 9 | 7 | 16 | 1 | 13 | 11 | 8 | 5 | 15 |
| Mean | 12.3 | 7.2 | 5.4 | 16.4 | 10.3 | 3.3 | 3.5 | 13.8 | 8.4 | 7.6 | 13.3 | 1.8 | 13.1 | 11.9 | 4.1 | 6.5 | 14.3 |
| Order of mean | 12 | 7 | 5 | 17 | 10 | 2 | 3 | 15 | 9 | 8 | 14 | 1 | 13 | 11 | 4 | 6 | 16 |

## Table 7: Rank of each country in each year for $CL^{C}_{(j),it}$

| Year | AUS | AUT | BEL | CAN | DNK | FRA | GER | HKG | IRE | ITA | JPN | NLD | SGP | ZAF | CHE | GBR | USA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1984 | 12 | 4 | 11 | 17 | 5 | 6 | 3 | 13 | 10 | 16 | 9 | 2 | 14 | 8 | 1 | 7 | 15 |
| 1985 | 11 | 4 | 10 | 17 | 9 | 5 | 3 | 13 | 12 | 14 | 8 | 2 | 15 | 6 | 1 | 7 | 16 |
| 1986 | 10 | 6 | 8 | 17 | 7 | 4 | 3 | 16 | 9 | 13 | 11 | 2 | 14 | 12 | 1 | 5 | 15 |
| 1987 | 13 | 9 | 7 | 17 | 3 | 8 | 4 | 14 | 10 | 12 | 6 | 2 | 16 | 11 | 1 | 5 | 15 |
| 1988 | 13 | 10 | 5 | 17 | 4 | 7 | 8 | 15 | 6 | 12 | 9 | 2 | 16 | 11 | 1 | 3 | 14 |
| 1989 | 14 | 12 | 3 | 17 | 4 | 5 | 8 | 15 | 6 | 10 | 9 | 2 | 11 | 13 | 1 | 7 | 16 |
| 1990 | 15 | 8 | 3 | 17 | 6 | 5 | 4 | 16 | 7 | 11 | 10 | 2 | 12 | 14 | 1 | 9 | 13 |
| 1991 | 15 | 9 | 2 | 17 | 11 | 3 | 5 | 14 | 7 | 6 | 10 | 4 | 12 | 16 | 1 | 8 | 13 |
| 1992 | 15 | 8 | 5 | 17 | 12 | 4 | 2 | 13 | 7 | 6 | 11 | 3 | 10 | 14 | 1 | 9 | 16 |
| 1993 | 15 | 6 | 5 | 17 | 10 | 4 | 3 | 14 | 7 | 9 | 11 | 1 | 12 | 13 | 2 | 8 | 16 |
| 1994 | 13 | 6 | 5 | 17 | 12 | 4 | 3 | 15 | 8 | 9 | 11 | 1 | 10 | 14 | 2 | 7 | 16 |
| 1995 | 14 | 6 | 5 | 17 | 10 | 4 | 2 | 12 | 7 | 9 | 15 | 1 | 11 | 13 | 3 | 8 | 16 |
| 1996 | 13 | 6 | 5 | 17 | 9 | 4 | 2 | 12 | 7 | 10 | 15 | 1 | 11 | 14 | 3 | 8 | 16 |
| 1997 | 13 | 7 | 4 | 14 | 10 | 6 | 2 | 9 | 5 | 17 | 15 | 1 | 16 | 12 | 3 | 8 | 11 |
| 1998 | 11 | 8 | 3 | 14 | 17 | 5 | 2 | 6 | 7 | 16 | 13 | 1 | 15 | 12 | 4 | 9 | 10 |
| 1999 | 11 | 6 | 4 | 16 | 17 | 5 | 2 | 10 | 7 | 12 | 14 | 1 | 15 | 9 | 3 | 8 | 13 |
| 2000 | 12 | 8 | 5 | 16 | 17 | 3 | 1 | 7 | 11 | 10 | 13 | 2 | 15 | 9 | 4 | 6 | 14 |
| 2001 | 11 | 10 | 8 | 15 | 17 | 3 | 2 | 6 | 12 | 7 | 16 | 1 | 14 | 9 | 4 | 5 | 13 |
| 2002 | 12 | 10 | 5 | 17 | 13 | 2 | 3 | 8 | 11 | 4 | 16 | 1 | 14 | 9 | 6 | 7 | 15 |
| 2003 | 12 | 10 | 7 | 17 | 9 | 2 | 3 | 11 | 8 | 6 | 16 | 1 | 15 | 14 | 4 | 5 | 13 |
| 2004 | 12 | 14 | 7 | 17 | 9 | 2 | 6 | 10 | 8 | 4 | 16 | 1 | 15 | 11 | 5 | 3 | 13 |
| 2005 | 10 | 12 | 4 | 17 | 9 | 1 | 7 | 15 | 8 | 3 | 16 | 2 | 14 | 11 | 5 | 6 | 13 |
| 2006 | 11 | 10 | 4 | 17 | 9 | 1 | 7 | 16 | 8 | 3 | 15 | 2 | 13 | 12 | 5 | 6 | 14 |
| 2007 | 10 | 11 | 4 | 17 | 8 | 1 | 6 | 15 | 9 | 3 | 14 | 2 | 13 | 12 | 7 | 5 | 16 |
| 2008 | 12 | 9 | 2 | 17 | 7 | 1 | 4 | 14 | 10 | 5 | 15 | 3 | 13 | 11 | 8 | 6 | 16 |
| 2009 | 12 | 9 | 5 | 17 | 8 | 1 | 6 | 14 | 10 | 3 | 16 | 2 | 13 | 11 | 7 | 4 | 15 |
| 2010 | 13 | 7 | 6 | 17 | 9 | 1 | 5 | 14 | 10 | 3 | 15 | 2 | 12 | 11 | 8 | 4 | 16 |
| 2011 | 13 | 6 | 7 | 17 | 9 | 1 | 4 | 14 | 10 | 3 | 15 | 2 | 12 | 11 | 8 | 5 | 16 |
| 2012 | 13 | 5 | 7 | 17 | 10 | 1 | 4 | 14 | 9 | 3 | 15 | 2 | 12 | 11 | 8 | 6 | 16 |
| 2013 | 12 | 7 | 4 | 17 | 11 | 2 | 3 | 14 | 8 | 5 | 15 | 1 | 13 | 10 | 9 | 6 | 16 |
| 2014 | 12 | 7 | 4 | 17 | 10 | 2 | 3 | 14 | 9 | 8 | 16 | 1 | 13 | 11 | 5 | 6 | 15 |
| 2015 | 12 | 7 | 5 | 17 | 11 | 2 | 3 | 14 | 9 | 8 | 16 | 1 | 13 | 10 | 6 | 4 | 15 |
| Mean | 12.4 | 8.0 | 5.3 | 16.7 | 9.8 | 3.3 | 3.8 | 12.7 | 8.5 | 8.1 | 13.2 | 1.7 | 13.3 | 11.4 | 4.0 | 6.3 | 14.6 |
| Order of mean | 12 | 7 | 5 | 17 | 10 | 2 | 3 | 13 | 9 | 8 | 14 | 1 | 15 | 11 | 4 | 6 | 16 |

**Table 8: Rank of each country in each year for $\mathbf{CL^{U}_{(j'),it}}$**

| Year | AUS | AUT | BEL | CAN | DNK | FRA | GER | HKG | IRE | ITA | JPN | NLD | SGP | ZAF | CHE | GBR | USA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1984 | 10 | 5 | 12 | 17 | 1 | 7 | 4 | 14 | 9 | 16 | 11 | 3 | 13 | 8 | 2 | 6 | 15 |
| 1985 | 11 | 3 | 10 | 17 | 8 | 6 | 4 | 13 | 12 | 16 | 9 | 5 | 14 | 2 | 1 | 7 | 15 |
| 1986 | 10 | 7 | 9 | 17 | 6 | 3 | 5 | 16 | 11 | 14 | 12 | 2 | 13 | 8 | 1 | 4 | 15 |
| 1987 | 12 | 6 | 7 | 17 | 5 | 8 | 3 | 16 | 9 | 13 | 10 | 2 | 14 | 11 | 1 | 4 | 15 |
| 1988 | 13 | 9 | 7 | 17 | 2 | 6 | 3 | 16 | 5 | 11 | 10 | 4 | 14 | 12 | 1 | 8 | 15 |
| 1989 | 13 | 11 | 7 | 17 | 2 | 5 | 4 | 16 | 6 | 9 | 10 | 3 | 14 | 12 | 1 | 8 | 15 |
| 1990 | 14 | 8 | 7 | 17 | 5 | 4 | 3 | 13 | 6 | 11 | 9 | 2 | 15 | 12 | 1 | 10 | 16 |
| 1991 | 12 | 7 | 4 | 17 | 10 | 5 | 3 | 15 | 8 | 6 | 11 | 2 | 14 | 13 | 1 | 9 | 16 |
| 1992 | 13 | 9 | 5 | 17 | 12 | 4 | 3 | 14 | 7 | 6 | 10 | 2 | 11 | 15 | 1 | 8 | 16 |
| 1993 | 14 | 6 | 3 | 17 | 10 | 5 | 4 | 13 | 7 | 8 | 12 | 1 | 11 | 15 | 2 | 9 | 16 |
| 1994 | 14 | 6 | 4 | 17 | 13 | 5 | 3 | 12 | 8 | 9 | 15 | 1 | 10 | 11 | 2 | 7 | 16 |
| 1995 | 13 | 4 | 5 | 17 | 9 | 6 | 2 | 14 | 8 | 10 | 15 | 1 | 11 | 12 | 3 | 7 | 16 |
| 1996 | 15 | 5 | 4 | 17 | 9 | 6 | 2 | 12 | 7 | 11 | 14 | 1 | 10 | 13 | 3 | 8 | 16 |
| 1997 | 12 | 3 | 5 | 17 | 9 | 7 | 2 | 10 | 6 | 16 | 13 | 1 | 11 | 15 | 4 | 8 | 14 |
| 1998 | 14 | 6 | 3 | 12 | 17 | 4 | 2 | 11 | 7 | 15 | 10 | 1 | 13 | 16 | 5 | 9 | 8 |
| 1999 | 14 | 6 | 3 | 16 | 17 | 4 | 2 | 11 | 8 | 10 | 13 | 1 | 15 | 9 | 5 | 7 | 12 |
| 2000 | 14 | 9 | 4 | 16 | 17 | 3 | 2 | 10 | 8 | 7 | 13 | 1 | 15 | 12 | 5 | 6 | 11 |
| 2001 | 13 | 8 | 7 | 14 | 17 | 3 | 2 | 12 | 9 | 6 | 15 | 1 | 16 | 10 | 4 | 5 | 11 |
| 2002 | 13 | 9 | 7 | 17 | 15 | 2 | 3 | 11 | 10 | 4 | 16 | 1 | 12 | 8 | 6 | 5 | 14 |
| 2003 | 14 | 10 | 7 | 17 | 13 | 2 | 3 | 9 | 8 | 4 | 16 | 1 | 15 | 11 | 5 | 6 | 12 |
| 2004 | 12 | 11 | 7 | 17 | 9 | 2 | 6 | 10 | 8 | 4 | 16 | 1 | 15 | 14 | 5 | 3 | 13 |
| 2005 | 10 | 11 | 5 | 17 | 8 | 2 | 7 | 12 | 9 | 4 | 13 | 1 | 14 | 15 | 3 | 6 | 16 |
| 2006 | 11 | 10 | 4 | 17 | 8 | 2 | 7 | 14 | 9 | 3 | 13 | 1 | 15 | 12 | 5 | 6 | 16 |
| 2007 | 11 | 10 | 4 | 17 | 8 | 1 | 7 | 15 | 9 | 3 | 14 | 2 | 13 | 12 | 5 | 6 | 16 |
| 2008 | 11 | 8 | 7 | 17 | 10 | 1 | 3 | 14 | 9 | 4 | 15 | 2 | 13 | 12 | 6 | 5 | 16 |
| 2009 | 12 | 8 | 4 | 17 | 10 | 1 | 5 | 14 | 9 | 3 | 15 | 2 | 13 | 11 | 7 | 6 | 16 |
| 2010 | 12 | 7 | 6 | 17 | 9 | 1 | 5 | 14 | 10 | 3 | 16 | 2 | 13 | 11 | 8 | 4 | 15 |
| 2011 | 12 | 6 | 7 | 17 | 8 | 2 | 4 | 14 | 10 | 3 | 16 | 1 | 13 | 11 | 9 | 5 | 15 |
| 2012 | 12 | 6 | 7 | 17 | 10 | 1 | 4 | 14 | 9 | 3 | 15 | 2 | 13 | 11 | 8 | 5 | 16 |
| 2013 | 12 | 5 | 7 | 17 | 10 | 2 | 3 | 14 | 9 | 4 | 15 | 1 | 13 | 11 | 8 | 6 | 16 |
| 2014 | 12 | 6 | 4 | 17 | 10 | 2 | 3 | 14 | 9 | 7 | 15 | 1 | 13 | 11 | 8 | 5 | 16 |
| 2015 | 12 | 5 | 6 | 17 | 10 | 2 | 3 | 14 | 9 | 8 | 15 | 1 | 13 | 11 | 7 | 4 | 16 |
| Mean | 12.4 | 7.2 | 5.9 | 16.7 | 9.6 | 3.6 | 3.6 | 13.2 | 8.4 | 7.8 | 13.2 | 1.7 | 13.2 | 11.5 | 4.2 | 6.3 | 14.7 |
| Order of mean | 12 | 7 | 5 | 17 | 10 | 2 | 3 | 13 | 9 | 8 | 14 | 1 | 14 | 11 | 4 | 6 | 16 |

**Figure 7: Multidimensional scaling among equity markets according to dissimilarity measure $1-\hat{\tau}_{ij}$ (data set two)**

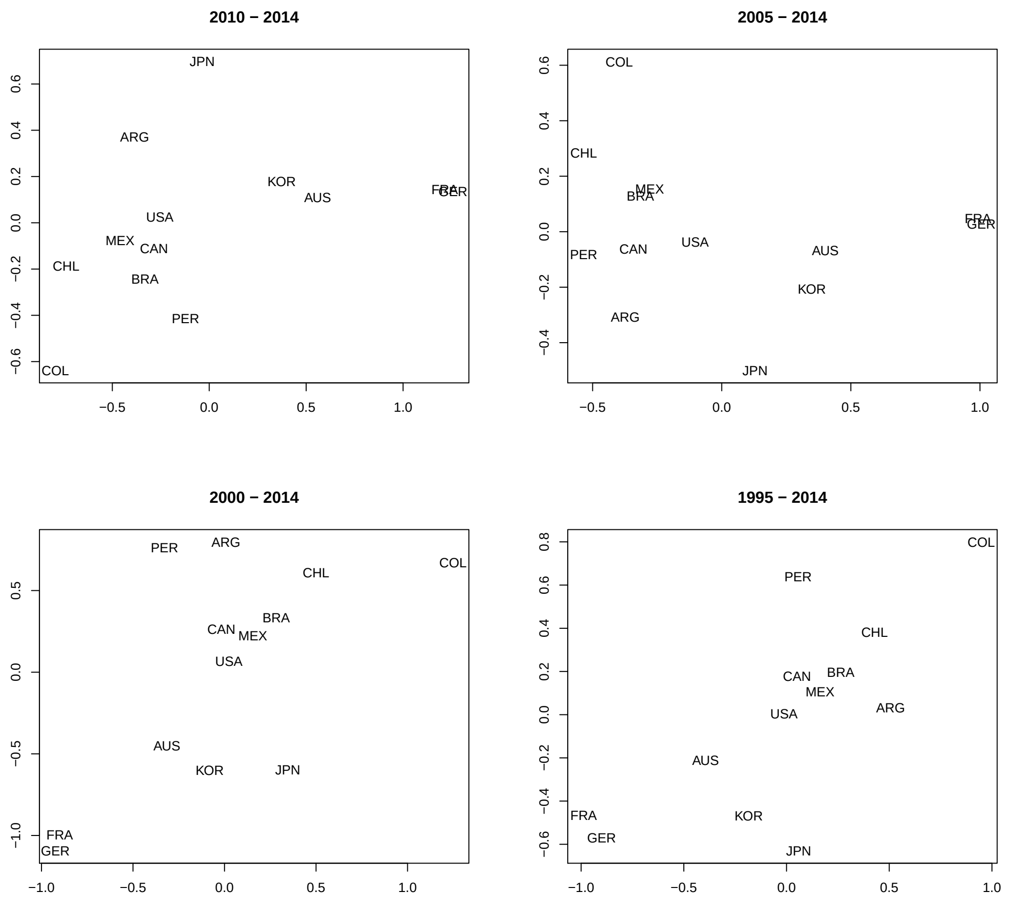
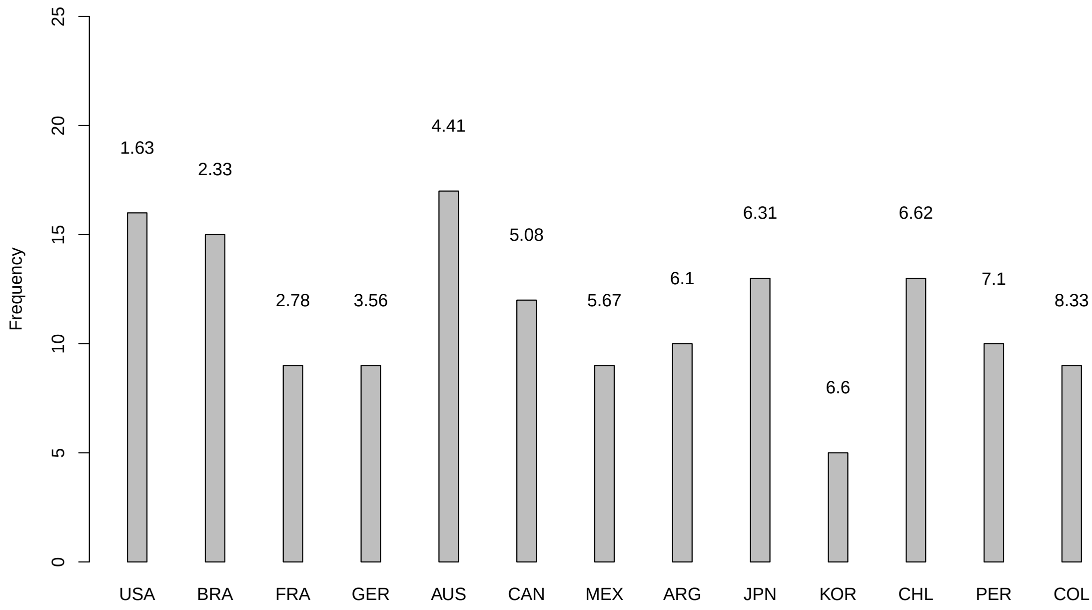


**Figure 8: Country specific relevance for interconnectedness among equity markets (data set two)**




Note: The height of each bar shows how frequently countries are chosen among the first $s$ root nodes over the 17 years. The number above each bar gives the average position each country has over time, given it is among the first $s$ root nodes

**Figure 9: Probabilities for rare comovements among financial markets over time (data set two)**

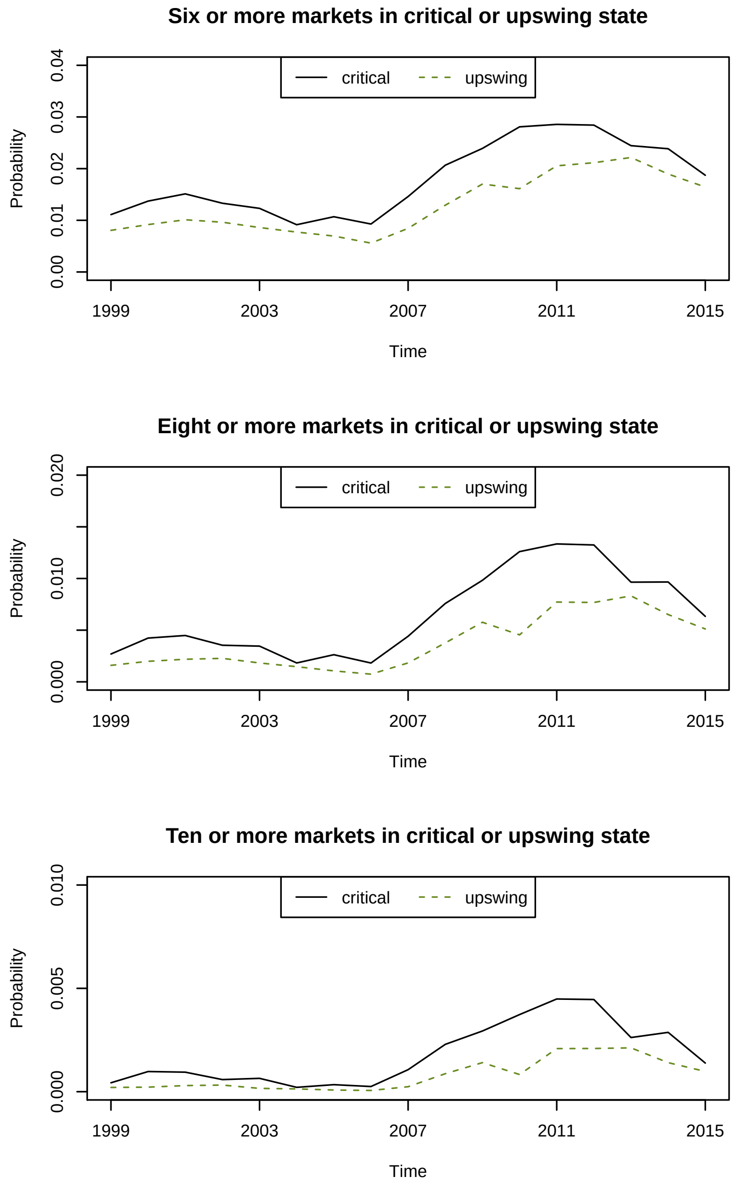


**Table 9: Rank of each country in each year for $\mathbf{CN^{C}_{(j'),it}}$ (data set two)**

| Year | MEX | COL | BRA | PER | CHL | ARG | JPN | GER | FRA | AUS | KOR | USA | CAN |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1999 | 3 | 13 | 4 | 8 | 9 | 2 | 12 | 6 | 10 | 7 | 11 | 1 | 5 |
| 2000 | 2 | 13 | 5 | 9 | 10 | 4 | 11 | 3 | 8 | 7 | 12 | 1 | 6 |
| 2001 | 2 | 13 | 4 | 7 | 8 | 3 | 9 | 10 | 11 | 6 | 12 | 1 | 5 |
| 2002 | 2 | 13 | 3 | 12 | 7 | 4 | 8 | 11 | 10 | 5 | 9 | 1 | 6 |
| 2003 | 2 | 13 | 5 | 12 | 6 | 11 | 7 | 9 | 10 | 3 | 8 | 1 | 4 |
| 2004 | 3 | 13 | 5 | 12 | 7 | 11 | 10 | 9 | 8 | 2 | 6 | 1 | 4 |
| 2005 | 4 | 13 | 6 | 12 | 9 | 11 | 10 | 8 | 5 | 2 | 7 | 1 | 3 |
| 2006 | 10 | 13 | 6 | 11 | 9 | 12 | 8 | 4 | 3 | 2 | 7 | 1 | 5 |
| 2007 | 3 | 13 | 1 | 10 | 6 | 9 | 12 | 8 | 7 | 5 | 11 | 2 | 4 |
| 2008 | 2 | 13 | 1 | 7 | 8 | 5 | 10 | 12 | 11 | 6 | 9 | 4 | 3 |
| 2009 | 2 | 10 | 1 | 7 | 8 | 5 | 9 | 13 | 12 | 6 | 11 | 4 | 3 |
| 2010 | 2 | 9 | 1 | 7 | 8 | 6 | 10 | 13 | 12 | 5 | 11 | 4 | 3 |
| 2011 | 3 | 11 | 1 | 6 | 7 | 5 | 8 | 13 | 12 | 10 | 9 | 2 | 4 |
| 2012 | 3 | 10 | 2 | 6 | 7 | 5 | 11 | 13 | 12 | 9 | 8 | 1 | 4 |
| 2013 | 4 | 10 | 3 | 6 | 5 | 7 | 11 | 13 | 12 | 8 | 9 | 2 | 1 |
| 2014 | 4 | 10 | 2 | 5 | 6 | 7 | 11 | 12 | 13 | 8 | 9 | 1 | 3 |
| 2015 | 4 | 8 | 3 | 5 | 6 | 12 | 10 | 13 | 11 | 7 | 9 | 1 | 2 |
| Mean | 3.2 | 11.6 | 3.1 | 8.4 | 7.4 | 7.0 | 9.8 | 10.0 | 9.8 | 5.8 | 9.3 | 1.7 | 3.8 |
| Order of mean | 3 | 13 | 2 | 8 | 7 | 6 | 10 | 12 | 10 | 5 | 9 | 1 | 4 |

## Table 10: Rank of each country in each year for $CN^{U}_{(j'),it}$ (data set two)

| Year | MEX | COL | BRA | PER | CHL | ARG | JPN | GER | FRA | AUS | KOR | USA | CAN |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1999 | 3 | 13 | 4 | 11 | 9 | 2 | 10 | 6 | 8 | 7 | 12 | 1 | 5 |
| 2000 | 2 | 13 | 5 | 11 | 6 | 3 | 10 | 9 | 8 | 7 | 12 | 1 | 4 |
| 2001 | 2 | 13 | 5 | 12 | 6 | 3 | 10 | 8 | 9 | 7 | 11 | 1 | 4 |
| 2002 | 2 | 13 | 4 | 12 | 7 | 5 | 11 | 10 | 9 | 6 | 8 | 1 | 3 |
| 2003 | 3 | 13 | 4 | 12 | 6 | 8 | 10 | 11 | 9 | 5 | 7 | 1 | 2 |
| 2004 | 2 | 13 | 6 | 12 | 8 | 11 | 9 | 10 | 7 | 4 | 5 | 1 | 3 |
| 2005 | 2 | 13 | 6 | 12 | 8 | 11 | 9 | 10 | 7 | 3 | 5 | 1 | 4 |
| 2006 | 3 | 13 | 4 | 12 | 10 | 11 | 8 | 9 | 5 | 2 | 7 | 1 | 6 |
| 2007 | 2 | 13 | 1 | 12 | 10 | 9 | 6 | 7 | 4 | 3 | 8 | 5 | 11 |
| 2008 | 3 | 13 | 1 | 11 | 12 | 7 | 5 | 8 | 6 | 4 | 9 | 2 | 10 |
| 2009 | 3 | 13 | 1 | 7 | 12 | 8 | 6 | 11 | 10 | 4 | 9 | 2 | 5 |
| 2010 | 3 | 11 | 2 | 5 | 7 | 8 | 9 | 12 | 10 | 6 | 13 | 1 | 4 |
| 2011 | 3 | 11 | 2 | 5 | 9 | 7 | 10 | 13 | 12 | 6 | 8 | 1 | 4 |
| 2012 | 4 | 12 | 2 | 5 | 7 | 9 | 8 | 13 | 11 | 6 | 10 | 3 | 1 |
| 2013 | 4 | 10 | 2 | 6 | 5 | 7 | 11 | 13 | 12 | 8 | 9 | 1 | 3 |
| 2014 | 4 | 10 | 3 | 6 | 5 | 7 | 11 | 13 | 12 | 8 | 9 | 1 | 2 |
| 2015 | 3 | 8 | 4 | 6 | 5 | 13 | 10 | 11 | 12 | 7 | 9 | 1 | 2 |
| Mean | 2.8 | 12.1 | 3.3 | 9.2 | 7.8 | 7.6 | 9.0 | 10.2 | 8.9 | 5.5 | 8.9 | 1.5 | 4.3 |
| Order of mean | 2 | 13 | 3 | 11 | 7 | 6 | 10 | 12 | 8 | 5 | 8 | 1 | 4 |

**Table 11: Rank of each country in each year for $\mathbf{CL^{C}_{(j),it}}$**

| Year | MEX | COL | BRA | PER | CHL | ARG | JPN | GER | FRA | AUS | KOR | USA | CAN |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1999 | 4 | 13 | 1 | 9 | 10 | 2 | 12 | 6 | 11 | 8 | 7 | 3 | 5 |
| 2000 | 2 | 13 | 3 | 9 | 10 | 4 | 11 | 5 | 8 | 7 | 12 | 1 | 6 |
| 2001 | 2 | 13 | 4 | 8 | 7 | 3 | 11 | 10 | 12 | 6 | 9 | 1 | 5 |
| 2002 | 3 | 13 | 2 | 10 | 5 | 1 | 9 | 12 | 11 | 7 | 8 | 4 | 6 |
| 2003 | 2 | 13 | 3 | 12 | 8 | 7 | 10 | 9 | 11 | 4 | 6 | 1 | 5 |
| 2004 | 2 | 13 | 3 | 11 | 7 | 8 | 9 | 12 | 10 | 4 | 5 | 1 | 6 |
| 2005 | 6 | 13 | 3 | 12 | 9 | 11 | 8 | 10 | 7 | 2 | 5 | 1 | 4 |
| 2006 | 6 | 13 | 1 | 12 | 8 | 11 | 9 | 10 | 7 | 4 | 5 | 2 | 3 |
| 2007 | 2 | 13 | 1 | 6 | 8 | 7 | 10 | 12 | 11 | 5 | 9 | 4 | 3 |
| 2008 | 2 | 11 | 1 | 7 | 8 | 6 | 10 | 13 | 12 | 4 | 9 | 5 | 3 |
| 2009 | 4 | 13 | 1 | 7 | 10 | 6 | 9 | 12 | 11 | 3 | 8 | 5 | 2 |
| 2010 | 2 | 11 | 1 | 6 | 8 | 7 | 10 | 13 | 12 | 4 | 9 | 5 | 3 |
| 2011 | 2 | 8 | 1 | 6 | 7 | 5 | 10 | 13 | 12 | 9 | 11 | 3 | 4 |
| 2012 | 3 | 11 | 2 | 6 | 7 | 5 | 10 | 13 | 12 | 9 | 8 | 1 | 4 |
| 2013 | 4 | 10 | 3 | 7 | 6 | 5 | 11 | 13 | 12 | 8 | 9 | 1 | 2 |
| 2014 | 4 | 10 | 2 | 5 | 7 | 6 | 11 | 13 | 12 | 8 | 9 | 1 | 3 |
| 2015 | 4 | 7 | 3 | 6 | 5 | 10 | 11 | 12 | 13 | 8 | 9 | 1 | 2 |
| Mean | 3.2 | 11.6 | 2.1 | 8.2 | 7.6 | 6.1 | 10.1 | 11.1 | 10.8 | 5.9 | 8.1 | 2.4 | 3.9 |
| Order of mean | 3 | 13 | 1 | 9 | 7 | 6 | 10 | 12 | 11 | 5 | 8 | 2 | 4 |

## Table 12: Rank of each country in each year for $\mathbf{CL^{U}_{(j'),it}}$

| Year | MEX | COL | BRA | PER | CHL | ARG | JPN | GER | FRA | AUS | KOR | USA | CAN |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1999 | 4 | 13 | 1 | 12 | 8 | 3 | 11 | 7 | 10 | 9 | 5 | 2 | 6 |
| 2000 | 2 | 13 | 3 | 11 | 9 | 4 | 10 | 6 | 7 | 8 | 12 | 1 | 5 |
| 2001 | 2 | 13 | 4 | 11 | 7 | 3 | 12 | 9 | 10 | 8 | 6 | 1 | 5 |
| 2002 | 4 | 13 | 3 | 11 | 6 | 1 | 12 | 10 | 9 | 8 | 7 | 2 | 5 |
| 2003 | 2 | 13 | 3 | 12 | 8 | 5 | 10 | 11 | 9 | 7 | 6 | 1 | 4 |
| 2004 | 2 | 13 | 4 | 12 | 9 | 7 | 8 | 11 | 10 | 6 | 3 | 1 | 5 |
| 2005 | 3 | 13 | 5 | 12 | 8 | 11 | 7 | 10 | 9 | 4 | 2 | 1 | 6 |
| 2006 | 3 | 13 | 1 | 11 | 7 | 9 | 8 | 12 | 10 | 5 | 6 | 2 | 4 |
| 2007 | 2 | 13 | 1 | 9 | 11 | 7 | 3 | 12 | 10 | 4 | 6 | 5 | 8 |
| 2008 | 2 | 13 | 1 | 8 | 10 | 6 | 5 | 12 | 11 | 4 | 7 | 3 | 9 |
| 2009 | 4 | 13 | 1 | 7 | 12 | 8 | 9 | 11 | 10 | 3 | 6 | 2 | 5 |
| 2010 | 3 | 11 | 2 | 5 | 9 | 7 | 8 | 13 | 12 | 6 | 10 | 1 | 4 |
| 2011 | 2 | 8 | 1 | 5 | 9 | 6 | 11 | 13 | 12 | 7 | 10 | 3 | 4 |
| 2012 | 4 | 13 | 2 | 6 | 8 | 9 | 10 | 12 | 11 | 5 | 7 | 1 | 3 |
| 2013 | 4 | 11 | 2 | 7 | 6 | 5 | 10 | 13 | 12 | 8 | 9 | 1 | 3 |
| 2014 | 3 | 10 | 2 | 6 | 7 | 5 | 11 | 13 | 12 | 8 | 9 | 1 | 4 |
| 2015 | 2 | 6 | 3 | 7 | 5 | 10 | 11 | 12 | 13 | 9 | 8 | 1 | 4 |
| Mean | 2.8 | 11.9 | 2.3 | 8.9 | 8.2 | 6.2 | 9.2 | 11.0 | 10.4 | 6.4 | 7.0 | 1.7 | 4.9 |
| Order of mean | 3 | 13 | 2 | 9 | 8 | 5 | 10 | 12 | 11 | 6 | 7 | 1 | 4 |